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Title: Rook Numbers and the Normal Ordering Problem

Abstract: For those familiar with chess, a rook is the piece that moves, and attacks, along rows and columns. The number of ways to arrange k rooks on the board in such a way that they don't attack each other is called the kth **Rook Number**. In the case where our board is a subset of an n by n square board, the rook numbers have to do with counting permutations of n elements with some restrictions. The case where our board is a **Ferrers board**, a.k.a. Young diagram, the rook numbers have some really nice properties.

And now, for something completely different. Let Ω be an algebra generated by two elements D, U, with a commutator relation $DU - UD = 1$. For $\omega \in \Omega$, the **Normal Ordering Problem** is to find the decomposition

$$\omega = \sum c_{i,j} U^i D^j,$$

where in each term, all the U's appear to the right of all the D's. This problem comes up in quantum mechanics, with U denoting the creation operator, and D the annihilation operator. It's easier to work with an operator where the D's act before U's, because, I suspect, it's easier to destroy than to create.

So how does a placement of non-attacking rooks relate to normal-ordering an operator? I'll show that, if the operator can be represented as a word, then the coefficients of its normal-ordered form are exactly rook numbers on a particular Ferrers board.