The meaning of vague definites: a Decision-Theoretic approach.*

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Abstract  In this paper, I am concerned with two mutually motivating goals. The empirical goal is to demonstrate that definite plurals are expressions whose interpretation requires an approach that incorporates goals of the speaker and hearer. The theoretical goal is to contribute to the development of a mathematical model of pragmatic reasoning involved in interpretation of vague utterances. I propose a Decision-Theoretic approach to vague definites in a variety of linguistic and situational contexts, and examine interaction of pragmatic and lexical cues to resolving vagueness.

Keywords: homogeneity presupposition, strongest meaning hypothesis, distributivity, maximality, Decision Theory, relevance, definite plurals

1 The problem

It’s been long noted that definite NPs tend to get exhaustive (narrow-existential) interpretations in negative contexts (1a), while also favouring exhaustive (universal) readings in non-negative sentences (1b).

(1)  Plural definites
    a. Peter didn’t see the linguists. (not any of the linguists)
    b. Peter saw the linguists. (all of the linguists)

Both the range of expressions that undergo this kind of variability and the factors that play a role in determining the final interpretation remain subject of current debate.

Schwarzschild (1994) notes the effect for definite plurals (1), and Lappin (1989) notes this for plural pronouns (2) and definite conjunctions (3).

(2)  Plural pronouns

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a. Peter didn’t see them. (none of them)
b. Peter saw them (all of them)

(3) **Definite conjunctions**

a. Peter didn’t take physics and algebra. (neither subject)
b. Peter took physics and algebra. (both subjects)

Löbner (2000) notes that this negation-related variability applies to predication in general (4), beyond just definites.

(4) **All predicates**

a. John didn’t walk to the store/read the book. (didn’t begin)
b. John walked to the store/read the book. (completed)

**Homogeneity Presupposition (HP) and the Strongest Meaning Hypothesis (SMH)** have been invoked as competing explanations for this monotonicity-dependent variability. Researchers investigating the interpretation of definite plurals, generic bare plurals, conjunctions, donkey conditionals with proportional quantifiers, bare (generic) conditionals (Löbner 1985, 1987, Schwarzschild 1994, Fodor 1970, Barker 1996, von Fintel 1997, Szabolcsi & Haddican 2004), among many others have turned to HP as the explanation for interpretive variability. A competing proposal, the SMH has been called upon to explain the interpretive variability in reciprocals, plurals (including conjunctions), and donkeys (Dalrymple, Kanazawa, Mchombo & Peters 1994, Dalrymple, Kanazawa, Kim, Mchombo & Peters 1998, Krifka 1996b, Winter 2001, Breheny 2005), calling into question both the descriptive validity and the explanatory depth of the HP for these phenomena. The Strongest Meaning Hypothesis can be thought of as the semantic (truth-conditionally relevant) version of the Gricean maxim of quantity (say as much as is required). As an instance of a general rationality-based reasoning, it has stronger explanatory appeal than the HP.

Here, I concentrate on the analysis of definite plurals, and demonstrate that the variability is only indirectly related to the monotonicity of these expressions’ environment. I argue that both HP and SMH fail to account for the full range of data. Instead, I propose that an improvement on the SMH can be made, if we tie the ultimate interpretation chosen for the NPs to the conversational goals of the speaker and hearer. The theoretical part of the paper then proposes a decision-theoretic approach to such an improvement.
2 Previous approaches

2.1 The Homogeneity Presupposition

HP, a variant of which is given in (5), has been used by Schwarzschild (1994) and Löbner (1985, 1987, 2000) to explain the data in (1), and extended by Szabolcsi & Haddican (2004) to apply to definite conjunctions (3).

(5) From Beck (2001) : $G$ is a plurality, $^*P$ is a pluralised, and thus distributive predicate.

\[ ^*P(G) = \begin{cases} 
1 & \text{if } \forall x(x \in G \rightarrow P(x)) \\
0 & \text{if } \forall x(x \in G \rightarrow \neg P(x)) \\
\text{undefined otherwise} & 
\end{cases} \]

Accounts that depend on HP run into several problems. First, Breheny (2005) points out that in examples like (6), there seems to be no expectation that the capture of the gangsters is an all-or-nothing matter, making homogeneity an implausible explanation for the preferred reading that the police, as of yet, caught none of them.

(6) The gang members have dispersed to the four corners of the world after the bank robbery, and the police haven’t found them yet.

Second, downward-entailing and NPI-licensing contexts other than negation also show the same interpretation preferences (7, from Breheny (2005)).

(7) If Peter beats the donkeys in his care, he will be prosecuted. (persecution for beating any donkey)

According to the HP, "Peter beats the donkeys in his care" presupposes that all of the donkeys get beaten (or none, if the sentence is false). Presuppositions of the antecedent project; thus the entire sentence should presuppose that either all or none of Peter’s donkeys are beaten. However, this prediction is not borne out - the sentence has a preferred reading in which Peter will be persecuted if he beats any of his donkeys. Thus, the Homogeneity Presupposition, which makes reference to negative and positive statements, cannot possibly derive the preference in (7), where other downward-entailing and NPI-licensing contexts pattern with negation. These reasons lead us to abandon the HP.

2.2 Strongest Meaning Hypothesis (SMH)

Krifka (1996b) proposes that when a predicate applies to any sum individual, the grammar does not specify how many subparts of the individual need to actually
satisfy the predicate, leaving the semantics vague. Then, the logically strongest available reading is chosen pragmatically (the Strongest Meaning Hypothesis, or SMH, 8) (Dalrymple et al. 1998; see also Beck 2001).

(8) The readings of semantically vague sentences are ordered by entailment relation. The strongest reading (one that entails all others) is preferred to others.

The SMH immediately explains why downward-entailing contexts favour the existential, and upward-entailing contexts the universal readings.

For example, in (7), the semantics leaves the conditional vague: Peter could be facing prosecution for beating some of the donkeys, or all of the donkeys. However, if he’s prosecuted for beating only some of the donkeys, all the more so for beating all of them: the non-maximal "any" reading entails the "all" reading. Therefore, this non-maximal reading is pragmatically preferred, according to the SMH.1 DE contexts will favour the non-maximal ("any") reading, while UE contexts will result the maximal ("all") reading for all predicates applying to sum individuals.

In the next section, I argue that the SMH fails to account for the context-dependent variability in definite plurals. The main reason for this failure is that it is not the strongest, but rather the most relevant readings that are pragmatically chosen for vague utterances.

2.3 Some under-appreciated data

The readings indicated in (1 - 4) are no more than tendencies, easily overridden when pragmatic context favours a different interpretation (9). This has been noted before in Krifka (1996b), Szabolcsi & Haddican (2004), and Breheny (2005), among others, but I think the gravity of this data has not been sufficiently appreciated.

In the context situation in (9a), the preferred interpretation for the utterance in (9b) is the opposite of the reading predicted by HP; the same context “flips” the preference for (9c).

1 Breheny (2005) claims that this proposal fails for specific indefinite plurals, such as (1). However, it is not clear why specific indefinites are problematic - they are, after all, sum individuals, so their semantic vagueness should be resolved by the SMH with no problem.

(1) Specific indefinite plurals
   a. Peter saw certain linguists. (he saw all of these linguists)
   b. Peter didn’t see certain linguists. (he failed to see any)
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(9) (example inspired by Krifka (1996b), (10))

a. **Situation:** Mary has a large house with over a dozen windows in different rooms. She locks up and leaves to go on vacation, forgetting to close just a few of the many windows in various rooms. A few minutes into the ride, her friend Max says, “There is a thunderstorm coming. Is the house going to be ok?” Mary replies:

b. **Utterance:** Oh my, we have to go back – the windows are open! (ok)

c. **Utterance:** Oh my, we have to go back – the windows aren’t closed! (ok)

Both HP and SMH fail to account for examples of pragmatic context-dependency like (9): the predication in those cases is non-homogenous, and it is the logically weaker meaning that is preferred.

Note, however, that the goals of conversational participants play a crucial role in achieving these interpretations, since changing the goals alters the choice of interpretation, in my example (10):

(10) a. **Situation:** Mary has a large house with over a dozen windows in different rooms. She is about to have the window-frames painted, and is preparing the house for the arrival of painters. Her friend Max says, “I think you still have a ton of work to do, right?” Mary replies:

b. **Utterance:** Actually, I’m done - the windows are open!

c. **Utterance:** Actually, I’m done - the windows aren’t closed (any more!)

With the new scenario and the same utterances, the judgments now conform to SMH predictions, in contrast to (9).

This is not an isolated example: consider another situation from Krifka (1996a), in (11) below. The natural interpretation, given the arrangement of doors and the goals of the speakers, is the non-maximal one (some of the doors), going against the predictions of SMH and HP.

(11) a. **Situation:** The local bank has a safe that is accessible only through a hallway with three consecutive doors (all of which must be open to reach the safe). Alice, a teller, would like to reach the safe, and her manager says:

b. **Utterance:** You need to get my key, the doors are closed.

c. **Utterance:** You need to get my key, the doors are not open.
Again, as with the previous examples, changing the goals of the interlocutors reverses the judgments to the maximal interpretation (some of the doors), and brings them into alignment with the SMH and HP. Thus, the most natural interpretation in my example (12) is that all of the doors are closed.

(12) a. **Situation**: The local bank has a safe that is accessible through a hallway with three consecutive doors. To set the alarm when the bank closes for the night, all of the doors must be closed - otherwise, the alarm cannot be turned on. Alice, a teller, is closing up for the night, and her manager says:
   
   b. **Utterance**: You can set the alarm now, the doors are closed.
   
   c. **Utterance**: You can set the alarm now, the doors are not open (any-more).

These examples demonstrate that interlocutors’ goals play a central role in selecting the preferred interpretation. So, perhaps we can predict the readings if we take these goals into account. In the remainder of this paper, I will develop and evaluate a proposal which extends a new line of research that has emerged over the last decade, seeking to incorporate pragmatic factors into a formal theory of natural language interpretation. In the next section, I review some of the pragmatic factors that must be included in interpreting definite plurals, and semantic foundations onto which these factors will be added. Subsequent section then make an initial Decision-Theoretic proposal that puts the pragmatics and semantics together.

3 Foundations for semantics and pragmatics of definite plurals

3.1 Vagueness in questions and plurals

Questions are often vague when considered out of context. For instance, the question in (13) is vague with respect to the level of detail required in the answer (the level of granularity), while the one in (14) is vague with respect to whether all or some of the possible answers should be listed in order to create a complete answer (the level of exhaustivity). Which degree of exhaustivity counts as a complete answer to a question has been subject to vigorous debate in the literature (?Berman 1991, Lahiri 1991, Heim 1994, ?, inter alia). Beck and Rullmann, among others, argue that both the maximal/exhaustive (mention-all) kinds of readings and also the non-maximal/non-exhaustive (mention-some) readings are available. Following Merin’s 1999 approach to relevance, van Rooy (1999) proposes a formal pragmatic account of question interpretation. In his analysis, questions are semantically underspecified or vague (13): their denotation depends on the goals and preferences of the questioner. In the context of these goals and preferences, vagueness is resolved: a partition is
chosen as the true import of the question (13b, 13c). If the goal of the questioner in (13) is to engage in small-talk about the hearer’s country, just giving the name of the country is sufficient (13b-i); however, this is not detailed enough if the goal is to mail a package to the hearer, and the full address must be given (13b-ii).

(13) **Vagueness in questions: granularity**
   a. Where do you live?
   b. **Possible partitions:**
      i. {You live in USA, You live in France, ...}
      ii. {You live at 1 Sue St. Enid OK, You live at 2 3rd St. Lodi NJ, ...}
   c. **Goal 1:** Questioner interested in background of hearer → partition i
      **Goal 2:** Questioner has to mail hearer a package → partition ii

(14) **Vagueness in questions: exhaustivity**
   a. Where can I buy an Italian newspaper?
   b. **Possible readings:**
      i. Provide an exhaustive list of all places selling Italian newspapers
      ii. Mention some place selling Italian newspapers
   c. **Goal 1:** Questioner wants to write a guidebook for Italian visitors to the city → reading i
      **Goal 2:** Questioner wants to read a newspaper in Italian → reading ii

Van Rooy’s approach derives two main types of vagueness: level of granularity as in (13), and degree of exhaustivity (14), where the desired answer could be strongly exhaustive (mention-all of the places selling Italian newspapers) or non-exhaustive (mention-some of the places).

Striking parallels exist between the types of vagueness inherent in questions and those in definite plurals. First, as repeatedly noted in the literature, definite plurals are vague with respect to the level of granularity (distributivity) (15).

(15) **Vagueness in definite plurals: distributivity ≈ granularity**
   a. The boys built a raft.
   b. **Possible interpretations:**
      i. Team1={Andy,Bill} built a raft and Team2={Chris,Dan} built a raft
      ii. Andy built a raft & Bill built a raft & Chris built a raft & Dan built a raft

Second, definites are vague with respect to exhaustivity: (16) is compatible with the maximal (all the windows) or non-maximal construal of the definite, e.g.
excepting a few closed windows if the hearer is wondering if his home is storm-proof (Krifka 1996b).

(16) **Vagueness in definite plurals: maximality \( \approx \) exhaustivity**

a. The windows are open.

b. **Possible interpretations:**
   i. All of the windows are open
   ii. Some of the windows are open

In resolving the vagueness, what factors affect the final interpretation for sentences with definite plurals? First, the nature of the VP plays a role. Many predicates require a certain level of distributivity or collectivity in the subject NP (17a, 17b) (Dowty 1987). At the same time, Yoon (1996) notes that some predicates seem to produce a lexical preference for maximal (17c) or non-maximal (17d) interpretation.

(17) a. The boys took a deep breath.
   b. The boys surrounded the castle.
   c. The children are healthy. The windows are closed.
   d. The children are sick. The windows are open.

While the VP clearly plays a role, it doesn’t fully determine the final interpretation – Krifka (1996b) argues that the preferences observed by Yoon are not lexically-driven, since they vary with situation. He gives (11), in which ‘closed’ (a total/maximal predicate, for Yoon) has a non-maximal interpretation. A point that Krifka does not make is that extra-linguistic factors play a role in resolving the vagueness in both distributivity and maximality.

(11) (11a) **Situation:** The local bank has a safe that is accessible only through a hallway with three consecutive doors (all of which must be open to reach the safe). Alice, a teller, would like to reach the safe, and her manager says:

(11b) **Utterance:** You need to get my key, the doors are closed.

Note that the contextual factors which influence the interpretation include, first, an actual salient arrangement of the entities within the NP into ‘packages’ (veggies pre-packaged in baskets, windows/doors on the outside of the house, doors in a sequence), but also, crucially, the goals of conversational participants. Schwarzschild (1991) notes the influence of packaging on distributivity with examples like (18a, from Schwarzschild (1991)).

To demonstrate the influence of interlocutors’ goals, I separate these goals and the arrangement of entities (their ‘packaging’) from the rest of the extra-linguistic
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context in (18, 19, 20) Thus, a grocery-store owner who has the goal in (18d) in the situation (18b) whose vegetables arrive as in (18c), will interpret the utterance (18a) to mean that all of the vegetables in the delivery, taken together, are too light for the big scale (collective interpretation, which is false in this situation).

In contrast, if the owner’s goal is the one in (18e), the same packaging and situation (18b, 18c) result in a very different interpretation for the same utterance (18a): specifically, the vegetables in each basket are too light for the big scale (intermediate distributive interpretation, which is true in this situation).

The contextual influences on examples in (9, 10, 11, 12) involving varying levels of maximality are spelled out similarly in (19, 20).

(18) Distributivity
a. **Utterance:** The veggies are too light for the big scale (excludes collective interpretation)
b. **Situation** The grocery has a big rough scale suitable for small truck-loads, and a small scale for weighing only a few veggies or fruits at a time.
c. **Packaging** Vegetables arrive at a grocery pre-packaged in baskets.
d. **Goal 1:** To double-check the amount delivered → Utterance is False
e. **Goal 2:** To price veggies for sale in baskets → Utterance is True

(19) Maximality
a. **Utterance:** The doors/windows are open
b. **Situation** Mary is the owner of a large house.
c. **Packaging** The house has six doors and a score of windows in its outside, on various sides along the perimeter.
d. **Goal 1:** To prepare the house for the arrival of window-frame or lintel painters → All doors/windows
e. **Goal 2:** To ensure the house is thief-proof during owner’s absence → Some doors/windows

(20) Maximality (continued)
a. **Utterance:** The doors are closed
b. **Situation** Alice is a teller at a local bank. Her manager has keys to all the doors. To set the alarm at the end of the day, all of the internal doors leading to the safe must be closed.
c. **Packaging** The bank safe is accessible only through a hallway with three consecutive doors (all of which must be open to reach the safe)
d. **Goal 1:** To reach the safe → Some door(s)
The empirical data thus indicates the conditions for a theory of interpretation of definite plurals. First, such a theory needs a vague or underspecified semantics that allows for the full range of the collective/distributive and maximal/non-maximal variation. Second, we need a way to encode the effects of the VP, the packaging of entities within the NP denotation, and the speaker/hearer goals in computing the truth-conditions. The idea that both packaging and speaker/hearer goals play a role in determining interpretation of definite plurals is fairly uncontroversial. Existing accounts, however, have either not spelled out the role of extra-linguistic factors, or focused on the role of packaging. My main theoretical contribution is to provide a way to include interlocutors’ goals in determining the interpretation of definite plurals. Next, I will sketch three promising approaches to distributivity and maximality in definite plurals, and point to the need for an upgraded account.

3.2 Prior approaches to distributivity and maximality: Landman (1989)

Building on the work of Link (1983), Landman (1989) offers a semantic theory that permits varying levels of distributivity and (non)maximality. He uses the star (*) operator to pluralize VPs, creating distributive interpretations, and permits predicates to distribute over sums of individuals (\(a \sqcup b \sqcup c\), but not over groups (\(\uparrow (a \sqcup b \sqcup c)\) or the team of \(a, b,\) and \(c\)). All non-maximal interpretations are instances of collective predication (where the plural is interpreted as a group). Thus, the interpretations for the sentences in (17a, 17b, ??) would arise from representations in (21), in a situation where Andy, Bill, Chris, and Dan are the boys, and \(d_1, \ldots, d_5\) are the doors (or windows) (following the arrow I include a simplified formula that indicates more precisely a situation in which the interpretation is true). Note that the only way to require a fully maximal interpretation is to make it fully distributive (21d).

(21) a. \([17a] = \text{[The boys took a deep breath]} = \\
\lambda w \exists e * \text{took.a.deep.breath}(e, w) \land *\text{Ag}(e) = \sigma x \ast \text{boy}(x, w) \Rightarrow \\
\Rightarrow (a \sqcup b \sqcup c \sqcup d) \in *\text{took.a.deep.breath}(w) \text{ (distributive)}\)

b. \([17b] = \text{[The boys surrounded the castle]} = \\
\lambda w \exists e * \text{surrounded.the.castle}(e, w) \land *\text{Ag}(e) = \uparrow \sigma x \ast \text{boy}(x, w) \Rightarrow \\
\Rightarrow \uparrow (a \sqcup b \sqcup c \sqcup d) \in *\text{surrounded.the.castle}(w) \text{ (collective)}\)

c. \([??, Goal2] = \text{[The doors are open]} = \\
\lambda w \exists e * \text{open}(e, w) \land *\text{Ag}(e) = \uparrow \sigma x \ast \text{door}(x, w) \Rightarrow \\
\Rightarrow \uparrow (d_1 \sqcup d_2 \sqcup d_3 \sqcup d_4 \sqcup d_5) \in *\text{open}(w)\)

2 I am allowing here, counter-intuitively, that predicates like ‘open’ or ‘take a breath,’ do not have a built-in lexical requirement that they must distribute fully down to individuals. Then, Landman (1989,
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d. \([11\text{ or } ??, \text{Goal}\ 1] = \text{[The doors are open]} = \lambda w \exists e * \text{open}(e, w) \& * \text{Ag}(e) \Rightarrow \sigma x * \text{door}(x, w) \Rightarrow (d1 \sqcup d2 \sqcup d3 \sqcup d4 \sqcup d5) \in * \text{open}(w) (\text{distributive})

It is not clear whether Landman’s original 1989 theory allows for the intermediate-distributive interpretations like (18, Goal 2), repeated below. However, his 1996 paper explicitly derives them via the adaptation of Schwarzschild’s 1991 proposal described in 3.4 and exemplified in 3.2, where artichoke, Brussels sprout, cauliflower, daikon, and endive are the vegetables. The definition of ‘cover-agent’ \(* \text{Ag}(e)\) is given in 3.4.

(21) e. \([18, \text{Goal}\ 2] = \text{[The veggies are too light for the big scale]} = \lambda w \exists e * \text{too} - \text{light}(e, w) \& * \text{Ag}(e) \Rightarrow \uparrow \sigma x * \text{veg}(x, w) \Rightarrow \uparrow (a \sqcup b) \sqcup (c \sqcup d \sqcup e) \in * \text{too} - \text{light}(w) (\text{intermediate-distributive})

Landman’s theory posits a pervasive ambiguity between singular and plural VPs, and does not distinguish between the maximal and non-maximal interpretations in the representation (cf. 21c and 21d). \(^3\) Even more importantly for us, as can be seen from (21), the theory does not provide an explanation of how and when the various interpretations arise, since it does not include an account of pragmatic factors (whether the structure of a situation or the intent of a conversational agent).

3.3 Existing approaches to distributivity and maximality: \textbf{Schwarzschild} (1991) and \textbf{Brisson} (1998)

Schwarzschild (1991) proposes an account of distributivity, focusing on spelling out the role of packaging by building a free variable over covers \((\text{Cov}_i)\) into the denotation of VPs, which allows them to distribute up to sub-pluralities of the definite plural given by the cover. A cover is defined in 22a; the specific cover in 22b

\(^{1996}\) can derive non-maximal interpretations for ‘open’ (or ‘take a breath’) by giving the windows or doors a collective responsibility for the state of being open. If instead we say that selectional requirements of ‘open’ and ‘take a breath’ demand full distributivity, then neither Landman (1989) nor the theory in Landman (1996) described in 3.4 below can derive non-maximal readings, and a pragmatic mechanism is necessary, like the one in Brisson (1998) or the one I propose below. \(^3\) Brisson (1998) brings this up as a problem for Landman’s approach: “Since on the groups approach nonmaximality is just a species of collectivity, there is no way in this approach to distinguish between a nonmaximal collective reading and a maximal collective reading... Since the groups theory gives us no way to distinguish between the two, it won’t help us discover the meaning of all.” Indeed, “all” forces maximality even with collective readings. However, this inability to distinguish between maximal and non-maximal interpretations in theory matches empirical data for collective predicates with plain definite plurals: since the plural agent bears collective responsibility for the action, we cannot distinguish whether all or some of the agent’s subparts actually participated. I will address the meaning of "all" later in this paper.
derives the intermediate-distributive interpretations (18, 22c) from the representation in (22d).

(22) a. Cover: a set of sets of entities, such that the union of the sets in the cover is the universe of discourse.
   b. \{\{art1,art2,B.sprout1,B.sprout2\},\{cauli,daik,endv\},\{John\...\}\}
   c. The vegetables are too heavy for the big scale and too light for the small scale.
   d. \(\forall x [x \in [Cov_i] \& x \subseteq [\text{the.vegetables}] \rightarrow x \subseteq [\text{too.heavy.sc1}&\text{too.light.sc2}]]\)

Brisson (1998) builds on Schwarzschild (1991) to permit exceptions by allowing mismatch between the distribution of individuals into the cover-cells and the NP denotations (ill-fitting covers, defined in 23a). Unlike Landman (1989), she only derives cases of a few salient exceptional items, such as the bathroom window (window 5) in (23b), which derives the interpretation for (23c) from the representation in (23d), rather than existential interpretations like (9b, ?? Goal 2).

(23) a. A cover is ill-fitting with respect to an NP denotation if some members of NP denotation are in the same cover-cell as non-members, so no union of cells in the cover equals the NP denotation.
   b. \{\{windw1\},\{windw2\},\{windw3\},\{windw4\},\{windw5,Mary,door\}\}
   c. The windows are open (but we didn’t get to the bathroom window yet).
   d. \(\forall x [x \in [Cov_i] \& x \subseteq [\text{the.windows}] \rightarrow x \subseteq [\text{open}]]\)

There are serious drawbacks in the cover-based account.

First, the final interpretation depends on the distribution of individuals into cover-cells, making non-maximality essentially a matter of narrowing the domain to a relevant set. In this, the cover-based account of Brisson (1998) inherits the problem of any theory that derives non-maximality through domain narrowing.

In fact, as examples (9b, ??) repeated below demonstrate, non-maximality is not a matter of domain selection of any kind: there is no “relevant set” of open (non-exceptional) windows, and no window is more relevant than any other – it is simply irrelevant which or how many windows are open, as long as some of them are. This problem is intrinsic in the cover-based approach to maximality, and derives incorrect interpretations for many non-maximal uses of definite plurals.

(9) Scenario: The house has a dozen windows on the outside. The hearer and speaker are going on a trip and want to make sure the house is safe.
   Utterance: The windows are open
   Cover-derived interpretation: The relevant/salient windows are open
   Actually: Some of the windows are open (it doesn’t matter which ones, and the speaker may not know which ones)
Second, the distribution of individuals into cover-cells is denoted by a free, deictic variable, whose denotation is fixed when a particular cover is made salient in preceding discourse or extra-linguistic context. This deictic nature of Cov, requires at least the speaker to know the exact composition of cover-cells, including the special cell containing exceptions, in the same way this is required for a pronoun.

This is strikingly contrary to fact: the speaker doesn’t need to know the identity of the vegetables for the intermediate distributivity interpretation to arise in (18, 22c), nor does he need to know the identity of the closed, exceptional windows for the non-maximal interpretation to obtain in (9b, ??).

One way to avoid making demands on speaker knowledge is to intensionalize the Cov variable, varying the distribution of individuals into cells with each possible world compatible with what the speaker knows. As Kratzer (forthcoming) notes, “amending Schwarzschild’s account of plural predication, we would want to say that plural predication depends on contextually provided cover functions, not just on contextually provided covers.” In terms of the needs outlined in the previous section, the cover account derives (at least some of) the influence of the VP (since the Cov variable is part of the VP). It also captures the “packaging” aspect of scenarios whenever context provides a salient cover that yields correct interpretation (thus, it fails to provide correct interpretation whenever such a salient cover is not available, for instance in the example of open windows (9)). However, cover-based approach still does not have a mechanism for integrating the influence of speaker/hearer goals into the analysis.

3.4 Landman (1996) and his recasting of Schwarzschild (1991)

In his wide-reaching article “Plurality”, Landman (1996) provides a recasting of Schwarzschild’s proposal in his own framework (what he dubs ‘Theory IV’ of plurality). The definite plural NPs have only group interpretations, and variations in their distributivity and maximality are achieved via a mechanism of cover roles. The cover roles are defined as in (24) and illustrated in the proof (25c) for the denotation (25b) for the sentence (25a), in a situation where Andy, Bill, Chris, and Dan are the boys (compare with 18, 22c).

(24) Definition: Let R be a thematic role. Then \( {\cdot} R \), the cover role based on R, is the partial function from \( D_e \) (domain of events) into \( D_d \) (domain of individuals) defined by

\[
{\cdot}R(e) = a \text{ iff } a \in \text{ATOM} \& \biguplus \{ \downarrow (d) \in \text{SUM} : d \in \text{AT}({\cdot}R(e)) \} = \downarrow (a)
\]

Paraphrase: a is either a singular atom or a group, and the set underlying a is the union of the elements underlying atomic subparts of the plural role based on R.
The cover roles are a very powerful mechanism: using them, we can derive the interpretations in (17a, 17b, ??) in a similar way (26). For instance, in the collectively-interpreted (26b), the (potentially plural) VP "surrounded the castle" in fact denotes a single atomic event. Its agent is "the boys" taken as a single group (impure atom) - a collective reading. Thus, when taken down to pure singularities (individual boys), the set underlying this agent is the union of all the atomic subparts of all the agents of surrounding-castle events. As it happens, since there is only one surrounding-castle event, the proof essentially says that when you take the set of boys underlying the agent of this event, you get the union of all such sets (i.e. you get that set back).

(25)  a. The boys built a raft (in a situation where there are two teams of boys).

b. \( \lambda w. \exists e \cdot \text{built.a.raft}(e, w) \land \forall e \cdot \text{Ag}(e) = \uparrow \sigma x \cdot \text{boy}(x, w) \Rightarrow \lambda w. \uparrow (a \sqcup b) \sqcup (c \sqcup d) \in \text{built.a.raft}(w) \)

c. \( e = f \sqcup g, \text{Ag}(f) = \uparrow (a \sqcup b), \text{Ag}(g) = \uparrow (c \sqcup d); \)

then \( \forall e \cdot \text{Ag}(e) = \uparrow (a \sqcup b) \sqcup (c \sqcup d) \)

\( \{ \downarrow (d) : d \in AT(*R(e)) \} = \{ a \sqcup b, c \sqcup d \}, \) and

\( \uparrow(\{ \downarrow (d) : d \in AT(*R(e)) \}) = \uparrow(\{ a \sqcup b, c \sqcup d \}) = a \sqcup b \sqcup c \sqcup d = \)

\( = \downarrow (\uparrow \sigma x \cdot \text{boy}(x, w)) \) q.e.d.

(26)  a. \([17a] = \[
\begin{align*}
\lambda w. & \exists e \cdot \text{took.a.deep.breath}(e, w) \land \forall e \cdot \text{Ag}(e) = \sigma x \cdot \text{boy}(x, w) \\
& e = f \sqcup g \sqcup h \sqcup i, \text{Ag}(f) = a, \text{Ag}(g) = b, \text{Ag}(h) = c, \text{Ag}(i) = d,
\end{align*}
\)

then \( \forall e \cdot \text{Ag}(e) = a \sqcup b \sqcup c \sqcup d, \)

so \( \uparrow(\{ \downarrow (d) : d \in AT(*R(e)) \}) = \uparrow(\{ a \sqcup b, c \sqcup d \}) = a \sqcup b \sqcup c \sqcup d = \)

\( = \downarrow (\uparrow \sigma x \cdot \text{boy}(x, w)) \) q.e.d.

b. \([17b] = \[
\begin{align*}
\lambda w. & \exists e \cdot \text{surrounded.the.castle}(e, w) \land \forall e \cdot \text{Ag}(e) = \sigma x \cdot \text{boy}(x, w) \\
e & \text{is atomic, } \text{Ag}(e) = \uparrow (a \sqcup b \sqcup c \sqcup d), \text{so } \uparrow(\{ \downarrow (d) : d \in AT(*R(e)) \}) = \uparrow(\{ a \sqcup b, c \sqcup d \}) = a \sqcup b \sqcup c \sqcup d = \downarrow (\uparrow \sigma x \cdot \text{boy}(x, w)) \) q.e.d.
\)

b. \([??, \text{Goal}2] = \[
\begin{align*}
\lambda w. & \exists e \cdot \text{open}(e, w) \land \forall e \cdot \text{Ag}(e) = \uparrow \sigma x \cdot \text{door}(x, w) \) (see footnote 2)
\end{align*}
\)

e is atomic, \( \text{Ag}(e) = \uparrow (d1 \sqcup d2 \sqcup d3 \sqcup d4 \sqcup d5) \),

so \( \uparrow(\{ \downarrow (d) : d \in AT(*R(e)) \}) = \uparrow(\{ d1, \ldots, d5 \}) = d1 \sqcup d2 \sqcup d3 \sqcup d4 \sqcup d5 = \downarrow (\uparrow \sigma x \cdot \text{door}(x, w)) \) q.e.d.

d. \([11] = \[
\begin{align*}
\lambda w. & \exists e \cdot \text{open}(e, w) \land \forall e \cdot \text{Ag}(e) = \sigma x \cdot \text{door}(x, w) \\
e & = f \sqcup g \ldots \sqcup j, \text{Ag}(f) = d1, \ldots, \text{Ag}(j) = d5, \\
\text{so } & \forall e \cdot \text{Ag}(e) = (d1 \sqcup d2 \sqcup d3 \sqcup d4 \sqcup d5) \\
\text{so } & \uparrow(\{ \downarrow (d) : d \in AT(*R(e)) \}) = \uparrow(\{ d1, \ldots, d5 \}) = d1 \sqcup d2 \sqcup d3 \sqcup d4 \sqcup d5 = \downarrow (\uparrow \sigma x \cdot \text{door}(x, w)) \) q.e.d.
Landman’s reinterpretation preserves the flexibility of Schwarzschild’s theory, allowing for the intermediate-distributive interpretations like (18, 22c, 25). Representation no longer contains the free, deictic variable \( Cov_i \).

On this approach, the speaker only has to know that there is a cover making the statement true (i.e., the speaker must only be sure of the existence of a proof like the one in (25c), where knowledge that the teams are boy-made will point to the existence of such a proof). The resulting analysis allows the full range of empirically attested interpretations, and says nothing about the way these various interpretations are derived and the factors that influence the derivation. Sentences like (27) just mean that subparts of the given group of windows/boys, in some packaging, are agents for (potentially plural) states of being open / events of raft-building.

(27)  
  a. The windows are open  
  b. The boys built a raft

This semantics is very weak. The only information transmitted with the use of a cover-role is that there exists some proof like the ones in (25, 26) – so, this incorrectly predicts that in a situation (11), repeated below, the speaker can say ‘The doors are open’ on the collective interpretation (26c), allowing some doors to be actually closed, since a proof exists as long as some doors are open.

(11)  
  Scenario: The hearer works in a bank and must get to a safe that can only be reached via three consecutive doors.  
  Utterance: The doors are open.  
  Interpretation allowed in Theory IV: Some/All of the doors are open  
  Actual interpretation: All of the doors are open

This is much less information than would actually be transmitted by such an utterance in the situation (11). Similarly, this account incorrectly predicts that in a situation from (18), repeated below, the speaker can truthfully utter ‘The vegetables are not too light for the big scale’ on the collective interpretation, even when the baskets are actually too light for the scale. After all, the possibility of putting all the baskets on the scale at once points to the existence of some way to make the sentence true.

(18)  
  Scenario: The vegetables arrive to the grocery store pre-packaged in baskets, and need to be priced by weight. The store has a scale suitable for only for truckloads of stuff.  
  Utterance: The vegetables are not too light for the big scale.  
  Interpretation allowed in Theory IV: The veggies, possibly weighed all at once, are not too light for the big scale  
  Actual interpretation: The veggies, weighed by the basket, are not too light for the big scale.
Thus, by getting rid of the deictic cover variable, we have lost crucial information in this case. While clearly too weak to stand on its own, this semantics is the perfect starting point for an account drawing on pragmatic factors to derive a stronger interpretation – after all, in a particular scenario, hearers are able to figure out whether all or some windows/doors are required to be open, and which packaging is meant when baskets of vegetables need to be weighed.

We can draw some interim conclusions from this discussion of existing approaches to vague definite plurals. In a theory of definite plurals, we must take into account that in accounting for the influence of packaging on interpretation, deixis to packaging is necessary in some cases (e.g., 18) – without it, the semantics is too weak, as shown above. At the same time, deixis to packaging is wrong in some cases (e.g., ??) – because it makes non-maximality a matter of domain selection, and so the interpretations derived using it are contrary to fact. We still need to incorporate speaker/hearer goals into the analysis. In the proposal I lay out in the next subsection, I will introduce a deictic variable that will always include reference to hearer’s goals, and will encode information about packaging only when such information is relevant to hearer’s goals. Subsequently, I will build on this proposal to better incorporate a model of conversational interaction into the pragmatic calculations.

3.5 A Decision-Theoretic proposal

I apply Merin’s (1999) and van Rooy’s 1999 Decision Theory (DT) based definition of relevance to definite plurals, replacing the cover analysis, integrating Landman’s 1996 recasting of Schwarzschild’s proposal with a principled account of how and when the various interpretations arise. The chief innovation is thus the unification of the vague/flexible semantics with a formal analysis of pragmatic factors influencing the truth-conditions of sentences with definite plurals. Agents in conversation are constantly modeling each other’s goals. As a part of cooperative communication, each speaker aims to change the hearers’ states of knowledge so as to help them progress towards their goals. This is the heart of the Cooperative Principle ?, and in particular, of the Relation (Relevance) maxim. When interpreting a vague utterance (in this case, one containing a definite plural), hearers select propositions which can influence their actions in achieving the goal. Agents’ goals (and more) can be represented as decision problems (DeP) they are solving. A DeP is a triple <P,U,A>, where the probability function P represents agent’s beliefs, utility function U reflects the agent’s preferences, and a set of (mutually exclusive) actions A the agent chooses from. A proposition q changes agent’s beliefs (P), resolving the DeP if, after q is learned, a single action has, in each resulting world, the highest utility.
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In making an utterance, the speaker aims to resolve hearers’ DeP⁴. A relevance ordering between propositions (28) yields the contextual criterion for licensing and choosing an interpretation for plural definites, just as it does for questions, with relevance defined as helpfulness in resolving the DeP.

(28) Proposition p is more relevant (better to learn) for resolving DeP than q (p > DeP q) iff
   i. p eliminates more actions as non-optimal than q does or
   ii. p eliminates the same number of actions as q does, and q entails p (i.e. q is over-informative)

This relevance ordering, built into the definition of the variable REL⁵ (29)⁶, allows the hearer to choose an appropriate interpretation for the vague definite.

(29) The relevance operator:  
Definition: REL(DeP)(VP)(NP)(w) =  
{ g : ↓{ ↓d : d ∈ AT(↓ g) } ⊆ ↓ NP( w )  
& ∃ h |↓{ ↓d : d ∈ AT(↓ h) } ⊆ ↓ NP( w )  
& [ λ w . VP( e , w ) & Ag(e) = h > DeP λ w . VP( e , w ) & Ag(e) = g ] ) }

Paraphrase: REL is a function that takes the decision problem, VP and NP denotations, and a world, and outputs a set of individuals g, which could be groups or sums.

The first conjunct (double-underlined) assures that when you boil g down to singularities, all those singular individuals are atoms in the NP denotation (e.g., ‘the boys’).

The second conjunct assures that no other such NP-made (e.g., boy-made) thing h is more relevant than g – as the underlined portion at the end states, there is no h such that it’s better to learn that “h VPs” than that “g VPs.”

The vague and flexible semantics assigning a cover-role to the NP is disambiguated by the pragmatic operator (hence, the use of the plural-roles, rather than cover-roles for the subparts g and h of the NP). REL replaces Schwarzschild’s (1991) Cov in encoding the vagueness and context-dependence. REL operator comes as part of the VP, following syntactic assumptions in Schwarzschild (1991) and preceding

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⁴ Or, rather, the speaker aims to resolve his/her estimate or hearer’s DeP. While practically all linguists agree that conversational participants represent each other’s goals and knowledge during a conversation, the processes and representations utilized during such modeling are subject of much ongoing research. For the purposes of this section, it is enough if speakers have an idea about the actions their hearers are choosing from, and can figure out the effect of learning various propositions on that action set.

⁵ REL for ‘relevant,’ instead of a similar variable Op proposed by van Rooy.

⁶ If x is an atom, ↓ x = x (Landman 1989). If x a sum, rather than an atom, I assume ↓ x = x.
literature on distributivity (30c presents the structure). The operator containing REL introduces a set of alternatives – $O_{Rel}$ is a function from VP and NP denotations to the set of optimally relevant propositions (30a). The sentential existential quantifier converts the set into a single proposition at the top. This last operation is the one introduced for the Hamblin semantics in Kratzer & Shimoyama (2002). The schema for the entire sentence is given in (30).

\[(30) \text{The relevance operator (continued):}
\]

a. $O_{Rel} = \lambda w.\lambda q.\lambda x.\lambda e.x \in REL(DeP)(P)(q)(w) \& P(e,w) \& ^*Ag(e,x)$

b. $\lambda w.\exists p[p \in \{\lambda v.[g \in REL(DeP)(VP)(NP)(v) \& VP(e,v) \& ^*Ag(e) = g]\} \& p(w) = 1]$

\[\text{‘There is a true proposition, taken from the set of optimally-relevant propositions saying that some subpart(s) of the NP do the VPing.’}\]

c. $IP^{op}$

$\exists$

$IP$

$\text{NP}$ $\text{VP}^{op}$ $\text{VP}$

$d. \ [VP^{op}] = [[O_{Rel} \ VP]] = \lambda w.\lambda q.\lambda x.\lambda e.x \in REL(DeP)(VP)(q)(w) \& VP(e,w) \& ^*Ag(e,x)$

\[I[IP] = [[NP_i \ VP^{op}]] = \lambda w.\lambda x.\lambda e.x \in REL(DeP)(VP)(NP)(w) \& VP(e,w) \& ^*Ag(e,x)

\[I[IP^{op}] = [[\exists IP]] = (30b) = \lambda w.\exists p[p \in \{\lambda v.[g \in REL(DeP)(VP)(NP)(v) \& VP(e,v) \& ^*Ag(e) = g]\} \& p(w) = 1]

These tools allow us to calculate the correct levels of maximality and distributivity for vague definite plurals. Here are a few worked-out examples to illustrate the schema in (30). First, to address (non)maximality, consider (31).7

\[(31) \text{a. Decision Problem: Before a thunderstorm, Hearer has to decide whether to go on with daily business (Action1) if all windows are closed or return home (Action2) if some windows are open.}

\[\text{b. Utterance: The windows are closed.}

\[= \lambda w.\exists p[p \in \{g(w) \in REL(DeP)(win)(closed)(w) \& g(w)\text{is closed}\} \& p = 1]\n
\[\text{c. Pool of propositions REL chooses from:}\]

7 In (31), the propositions with a single underlining don’t resolve the DeP, and are eliminated by REL.
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\[ w_1 \sqcup w_2 \sqcup w_3 \text{ is closed} \]
\[ \uparrow (w_1 \sqcup w_2) \sqcup w_3 \text{ is closed} \]
\[ \uparrow (w_1 \sqcup w_2) \sqcup \uparrow (w_1 \sqcup w_3) \text{ is closed} \]
\[ \uparrow (w_1 \sqcup w_2) \sqcup \uparrow (w_1 \sqcup w_3) \sqcup w_4 \sqcup w_5 = \ast \text{Ag}(e) \} \]
\[ \& p = 1 \]
\[ \text{pool of propositions REL chooses from: (see footnotes 2, 6, 7, 8)} \]

Consider an agent facing the problem in (31a). If he hears (31b), he has to select one of the interpretations [i] or [ii] as the true import of the vague literal statement. While [i] resolves the decision problem (by pointing to Action1), [ii] fails to do so, since it is compatible both with a scenario where all windows are closed, and one in which some are open and some closed. So, [i] is the only relevant interpretation.

A built-in consequence of the framework is this: the only way to force a maximal interpretation is for REL to produce a single most-relevant proposition, so that the existential quantification over propositions is over a singleton set. Further, we inherit from Landman the fact that maximal interpretations are simply fully distributive ones, where a unique optimal proposition involves distributive predication over a sum. To illustrate what happens when these conditions fail to hold, consider the example in (32) below.\(^8\)

(32) a. **Decision Problem:** Hearer is preparing his house for arrival of painters, who will paint all the window-frames. He has to decide if he can relax till they arrive (Action 1), or if he still needs to do something to prepare for the painters (Action 2).

b. **Utterance** - same as in (31b): The windows are closed

\[ \lambda w. \exists p[p \in \{ g(w) \in REL(DeP)(w)(closed)(w) \& g(w) \text{ is closed} \} \& p = 1] \]

c. **Pool of propositions REL chooses from:** (see footnotes 2, 6, 7, 8)

---

\(^8\) Propositions with double underlining are over-informative and so are eliminated by REL.
Consider an agent facing a different problem (32a). If he hears the same utterance as before (31b, 32b), then both [i] and [ii] resolve the DeP (by pointing to Action2) and thus are both relevant. But [i] entails [ii] and thus is over-informative (hence less relevant according to (28)). This points to [ii] as the intended message. Thus, whenever REL fails to winnow the set down to a single fully-distributive proposition, the existential quantification over propositions will result in a weaker meaning, stating that one of the most-helpful propositions is true. In the case when the hearer doesn’t care which windows are closed, all propositions stating that one of the windows is closed are equally most-helpful. Thus, the existential quantification over propositions creates the effect of existential quantification over the windows, i.e., the non-maximal reading.

4 Relevance and vagueness in complex sentences

4.1 Downward-entailing contexts

Returning to the data in the first section, we must account for the tendency of maximality judgments to switch under negation, and in other downward-entailing contexts.

(1)  (1a) Peter didn’t see the linguists. (not any of the linguists)
    (1b) Peter saw the linguists. (all of the linguists)

(33)  a. I doubt that the children will understand the story. (I doubt any of them will)
    b. I believe that the children will understand the story. (I believe all of them will)
In selecting the most relevant children or linguists, the hearers must consider the entire utterance context (including negation or doubt), and not just the basic clause composed of the predicate VP and the agent NP. Thus, the Decision-Theoretic relevance operator in (29) must be minimally revised, enlarging the propositions in the pool to include the entire utterance, and allowing non-agent NPs, such as the object in (1) to be resolved.

In order to "re-constitute" the entire sentence while keeping track of the predicate-argument pair whose vagueness is being resolved, we use indexing: the predicates (verbs, VPs, or adjectives) are numbered, and the theta-role of the argument whose subparts are in question is indicated. A sentence containing a vague predicate-argument pair is co-indexed with such a pair: I use $n$ as a variable over numbers, and $\theta$ as a variable over theta roles; thus $X_n\theta_n$ is a syntactic object $X$ (usually, an IP) which contains a predicate with the index $n$ and an NP to which that predicate assigns the theta role $\theta_n$ (for perspicuity, I will index the NP with the theta-role assigned to it, $NP_{\theta_n}$). Thus, the interpretation $[X_n\theta_n]$ of such a syntactic object contains $[\text{Predicate}_n](e,w)\&^\theta_n(e) = [NP_{\theta_n}]$. In constructing the pool of potentially-relevant propositions, we substitute various combinations of the subparts from the denotation of the $NP_{\theta_n}$ (h and g in (29')) into that NP’s slot in $[X_n\theta_n]$. I use the notation $[X][h/[NP]]$ to indicate the denotation of $X$ in which the object $h$ substitutes for the denotation of the NP. (29') The relevance operator:

**Definition:** $REL(DeP){(X_n\theta_n)}(w) = \{g : \sqcup\{d : d \in AT(\sqcup g)\} \subseteq \downarrow [NP_{\theta_n}](w) & \neg \exists h[\sqcup\{d : d \in AT(\sqcup h)\} \subseteq \downarrow [NP_{\theta_n}](w) & [\lambda w.[X_n\theta_n][h/[NP_{\theta_n}]] >_{DeP} \lambda w.[X_n\theta_n][g/[NP_{\theta_n}]]\}] =$ spelling out the propositions whose relevance is being compared $= \{g : \sqcup\{d : d \in AT(\sqcup g)\} \subseteq \downarrow [NP_{\theta_n}](w) & \neg \exists h[\sqcup\{d : d \in AT(\sqcup h)\} \subseteq \downarrow [NP_{\theta_n}](w) & [\lambda w.[...[\text{Predicate}_n](e,w)\&^\theta_n(e) = h...] >_{DeP} [\lambda w.[...[\text{Predicate}_n](e,w)\&^\theta_n(e) = g...]]\} =$

**Paraphrase:** $REL$ is a function that takes the decision problem, a proposition containing a predicate and its argument NP denotations, and a world, and outputs a set of individuals $g$, which could be groups or sums. The first conjunct (double-underlined) assures that when you boil $g$ down to singularities, all those singular individuals are atoms in the NP denotation.

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9 I remain neutral on the important syntactic question of whether adjectives can assign syntactic theta-roles. Instead, I use the term to denote semantic roles that semantic arguments play in eventualities (both states and events).
(e.g., ‘the children’).

The second conjunct assures that no other such NP-made (e.g., child-made) thing \( h \) is more relevant than \( g \) – as the single-underlined portion states, there is no \( h \) such that it would be better to learn the proposition \( [X] \) about \( h \) fulfilling the predicate, than to learn the proposition \( [X] \) about \( g \) fulfilling the predicate.

This modification is able to derive the contextually-appropriate interpretations of (1, 33) in a manner fully parallel with the examples in the previous section.

Now that the expressions considered by REL are entire utterances, a technical question arises in cases of predicate coordination, especially when different levels of maximality or distributivity of the same NP are involved for the two predicates, as in (34): how are the propositions for REL constructed? And how are the results of the relevance calculation combined to give the ultimate reading that is optimally relevant for both predicates?

(34) **Coordinated predicates**

a. **Situation:** The owner of a house has not occupied or seen it for a year. Soon, there will be guests arriving to the house. To prepare, the house must be in good repair (nothing broken), and the rooms must be aired (by opening some windows). The speaker is a friend of the owner, who goes to check the situation, and to determine which of the four actions must be chosen: \( a_1 = \) repair broken windows and open some, \( a_2 = \) open some windows (no repairs needed), \( a_3 = \) repair broken windows (some are already open), \( a_4 = \) do nothing (no repairs needed, and the place is being aired).

b. The windows are open and unbroken.

c. The windows are closed but broken.

Since the subparts of the NP involved in the two predicates may be different, the sets of the most relevant such subparts must be calculated separately for the two predicates. In constructing the propositions from which the REL chooses optimal ones for each predicate, the other predicate will remain vague. The definition of REL in (29′) will produce a separate calculation for each indexed predicate-vague NP pair, constructing for each predicate its own set of maximally-relevant subparts of the NP. The pool of propositions for each calculation is constructed separately by cycling through different subparts of the NP with respect to one predicate, then the other (35 below).

Once the two sets of optimal NP-subparts are chosen by REL, they must then be re-incorporated to achieve the precise contextually-relevant interpretation for the
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whole utterance. The operator $O_{Rel}$ (30a) must be modified to achieve this, by taking the sets of optimal subparts of the NP, and combining these pointwise with the correct predicates, to generate the final set of propositions. The top existential quantifier will operate on this set of propositions, as before. Since $O_{Rel}$ is an operator that triggers the relevance calculation which considers the whole utterance, it must be introduced at the top of the sentence. The operator takes a proposition containing vague NP and predicate denotations, and creates a set of propositions that are simultaneously optimally-relevant with respect to each NP-predicate pair in the utterance. This is done by plugging in the elements from each predicate’s REL-set of optimally relevant subparts of the NP into the part of the total proposition where that predicate’s theta role is assigned (30a'). Since the REL-sets of optimally relevant subparts of the NP are indexed to the theta-role of the corresponding predicate, they are matched to that theta-role the propositions. In the structural representation, the vague NP-predicate pairs become visible to $O_{Rel}$ through co-indexing. The new structure of an example proposition is shown in (30c') below.

Consider now the interpretation of the sentence (34b), whose structure is given in (35a) in the situation (34a); for simplicity I assume there are only two windows, d1 and d2. The lower IP is interpreted as a vague proposition, containing two predicate-NP pairs: the windows are open, and the windows are unbroken (35b). These pairs are coindexed with the relevance operator $O_{Rel}$, triggering the calculation of two different sets of windows - those which are optimally relevant with respect to being open (35c), and those which are relevant with respect to being unbroken (35d). These sets are then used in constructing the set of optimally-relevant propositions, according to the definition in (30a'), with the elements of the set of windows most
relevant for "open" assigned the theta-role by "open", and the windows from the set most relevant for "unbroken" assigned the theta-role by "unbroken." The resulting set of propositions is therefore optimally relevant with respect to both predicates; the existential at the top turns this into a single proposition stating that at least one of the propositions in the set is true (35e).

(34b) The windows are open and unbroken.

(35) a.

\[ IP^{\text{top}} \]

\[ \exists \]

\[ O_{\text{Rel}}^{1\theta,2\theta_2} \]

\[ IP_{1\theta_1,2\theta_2} \]

\[ \text{NP} \]

The windows

\[ \text{are} \]

\[ \text{Pred}_1 \text{ and} \]

\[ \text{Pred}_2 \text{ \unbroken} \]

| open |
| unbroken |

b. \[ [IP_{1\theta_1,2\theta_2}] = \lambda w. \ast \text{open}(e_1, w) \& ^c \theta_1(e_1) = \uparrow \sigma x \ast \text{windw}(x, w) \& \ast \text{unbroken}(e_2, w) \& ^c \theta_2(e_2) = \uparrow \sigma x \ast \text{windw}(x, w) \]

e. \[ [IP^{\text{top}}] = \lambda w \exists p [p \in \{ \]

\[ \lambda w. \ast \text{open}(e_1, w) \& ^c \theta_1(e_1) = d_1 \& \ast \text{unbr}(e_2, w) \& ^c \theta_2(e_2) = d_1 \& d_2, \]

\[ \lambda w. \ast \text{open}(e_1, w) \& ^c \theta_1(e_1) = d_2 \& \ast \text{unbr}(e_2, w) \& ^c \theta_2(e_2) = d_1 \& d_2, \]

\[ \lambda w. \ast \text{open}(e_1, w) \& ^c \theta_1(e_1) = d_1 \& d_2 \& \ast \text{unbr}(e_2, w) \& ^c \theta_2(e_2) = d_1 \& d_2 \}

\[ \& p(w) = 1 \]

In this situation, as long as some of the windows are open, the actions which include opening the windows are eliminated (35c). Thus, any proposition stating
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that a window is open is equally relevant, resulting in several alternative propositions corresponding to different subsets of windows (35e). As a result, the windows are non-maximal with respect to being open. However, a statement that some of the windows are unbroken is compatible both with a situation requiring repairs, and also with a situation not requiring them, leaving the Decision Problem completely unresolved (35d). Only the propositions involving all of the windows being unbroken tell us that no repairs are needed - this is the optimally-relevant option reflected in each of the resulting propositions in (35e). Thus, the windows are maximal with respect to being unbroken.

Finally, combining negation and predicate coordination, consider the sentence in (36), in the context of the decision problem repeated from (34a). On the reading where negation has scope over conjunction, the sentence has the same interpretation as the one in (??), since English obeys DeMorgan’s law. Note that the earlier system (29), in which the propositions in the pool evaluated by REL were "local" in the sense of including just the NP and the predicate, would encounter a scopal paradox in this reading: on the one hand, negation scopes over the conjunction of the two predicates, while on the other hand, the negation must be included in the relevance calculation for each conjunct. The proposal in (29′) in which the entire utterance is considered avoids this problem.

(34a) **Situation:** The owner of a house has not occupied or seen it for a year. Soon, there will be guests arriving to the house. To prepare, the house must be in good repair (nothing broken), and the rooms must be aired (by opening some windows). The speaker is a friend of the owner, who goes to check the situation, and to determine which of the four actions must be chosen: a1 = repair broken windows and open some, a2 = open some windows (no repairs needed), a3 = repair broken windows (some are already open), a4 = do nothing (no repairs needed, and the place is being aired).

(36) **Negating coordinated predicates**
The windows aren’t open and unbroken.

(37) closedorbr The windows are closed or broken.

The derivation for the negation-over-conjunction reading is very similar to that in (35), and includes the negation in calculating the most-relevant windows (38).
In cases when vague NPs are present in more than one position, we can form pairs of NP-denotations, and consider the entire vague utterance with simultaneous

This Decision-Theoretic approach to maximality and distributivity naturally extends to cases involving vague definite plural NPs in two different roles, as in (39). In cases when vague NPs are present in more than one position, we can form pairs (tuples) of NP-denotations, and consider the entire vague utterance with simultaneous
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precisifications of the two NPs (40). The \(O_{Rel}\) operator then goes through this set of pairs, matching the optimal subparts of the two NPs to the appropriate theta-roles (41).

\[(39)\] The students interviewed the professors.

\[(40)\] **Extended definition:** \(REL(DeP)(X_{n\theta_i^j\theta_i^j})(w) = \{ < g_i, g_j > : \bigcup \{ d : d \in AT (\downarrow g_i) \} \subseteq [NP_{\theta_i^j}](w) \}
\]

\& \& \{ d : d \in AT (\downarrow g_i) \} \subseteq [NP_{\theta_i^j}](w)

\& \& \{ d : d \in AT (\downarrow g_j) \} \subseteq [NP_{\theta_i^j}](w)

\& \& \{ \lambda w.\llbracket X_{n\theta_i^j\theta_i^j} \rrbracket [h_i/\llbracket NP_{\theta_i^j} \rrbracket , h_j/\llbracket NP_{\theta_i^j} \rrbracket ] > DeP

\& \& \lambda w.\llbracket X_{n\theta_i^j\theta_i^j} \rrbracket [g_i/\llbracket NP_{\theta_i^j} \rrbracket , g_j/\llbracket NP_{\theta_i^j} \rrbracket ] > DeP

Spelling out the single-underlined part, the propositions whose relevance is being compared, the second conjunct makes sure that there is no \(h_i, h_j\) such that:

\[
[\lambda w.\llbracket \ldots [Predicate_e] \rrbracket (e, w) & \& \theta^i_n(e) = h_i & \& \theta^i_n(e) = h_j \ldots ] > DeP
\]

\[
[\lambda w.\llbracket \ldots [Predicate_e] \rrbracket (e, w) & \& \theta^i_n(e) = g_i & \& \theta^i_n(e) = g_j \ldots ]
\]

\[(41)\] \(O_{Rel}^{n\theta_i^j\theta_i^j} = \lambda w.\lambda \phi. \lambda x_i. \lambda x_j. \lambda e. \phi = \llbracket X_{n\theta_i^j\theta_i^j} \rrbracket \&
\]

\(< x_i, x_j > \in REL(DeP)(X_{n\theta_i^j\theta_i^j})(w) \&
\]

\& \[\llbracket X \rrbracket [x_i/\llbracket NP_{\theta_i^j} \rrbracket , x_j/\llbracket NP_{\theta_i^j} \rrbracket ] = \]

spelling out the contents of the combined proposition

\[
= \lambda w.\lambda \phi. \lambda x_i. \lambda x_j. \lambda e. \phi = \llbracket X_{n\theta_i^j\theta_i^j} \rrbracket \&
\]

\(< x_i, x_j > \in REL(DeP)(X_{n\theta_i^j\theta_i^j})(w) \&
\]

\& \[\llbracket X \rrbracket [x_i/\llbracket NP_{\theta_i^j} \rrbracket , x_j/\llbracket NP_{\theta_i^j} \rrbracket ] = \]

\& \[\ldots [Predicate_e] (e, w) & \& \theta^i_n(e) = x_i & \& \theta^i_n(e) = x_j \ldots ]

For the concrete example in (39), suppose that the context involves the student reporters Alice and Bob interviewing the two award-winning professors C. and D. for the school newspaper - a task that would be accomplished in case if both C. and D. are interviewed, by either of the two students or by both of them together. The desired reading is thus non-maximal on students, and maximal on professors, and the optimal interviewer-interviewee pairs are in (42b). The definition in (41) will yield the set of propositions in (42c), generating the desired reading.
This system is not sufficient for situations in which, for instance, Alice and C. are in the math department, while Bob and D. are in the linguistics department, and the students’ task is to interview the award-winning professor in their respective departments. In order to generate such readings, each student-professor pair has to be tied to a particular subevent of the plural event of interviewing, the way subparts of NPs are tied to subparts of events in the proofs in Landman (1996) (section 3.4). I leave the mechanics of this extension to future work.

5 Conclusions

5.1 Back to questions

The DT approach takes as its base a weak semantics for definite plurals, and builds in pragmatic factors to derive stronger truth-conditions for sentences with definites. Speakers’ estimates for each other’s beliefs, goals, and available actions are incorporated into selection of maximally relevant propositions with subparts of the plural definite. The framework derives the (non)maximality and distributivity patterns and makes correct predictions about speaker knowledge.
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This framework makes interpretation of definite plurals parallel to that of questions (van Rooy 1999), where speakers’ estimates for each other’s Decision Problems are incorporated into selection of the set of maximally relevant answers. Thus, in (13a), repeated below as (43), the level of granularity is chosen based on the decision problem - the problem in (43d-i) yields the reading corresponding to the partition in (43b-i), while the problem in (43d-ii) results in the hearer choosing the partition in (43b-ii) as the meaning of the question.

(43) a. Where do you live?

b. Possible partitions:
   i. {You live in USA, You live in France, ...}
   ii. {You live at 1 Sue St. Enid OK, You live at 2 3rd St. Lodi NJ ...}

c. \{\lambda v[g \in REL(place)(P)(v)] : w \in W & g \in REL(place)(P)(w)\},
   where \(P = \lambda w.\lambda x.\) You live in \(x\) in \(w\)

d. i. Decision Problem1: Questioner interested in background of hearer to select a conversation topic: US sports (Action1), French cuisine (Action2), ...
   ii. Decision Problem2: Questioner has to mail hearer a package (actions corresponding to different addresses)

(44) a. Where can I buy an Italian newspaper?

b. Possible sets:
   i. Mention-all: \{\{u = \text{It. news sold at the palace}, \{v = \text{It. news sold at the station}\}, \{w = \text{It. news sold at the palace & the station}\}\}
   ii. Mention-some: \{\{u, w\}, \{v, w\}\}

c. \{\lambda v[g \in REL(place)(P)(v)] : w \in W & g \in REL(place)(P)(w)\},
   with \(P = \lambda w.\lambda x.\) Italian newspaper sold at \(x\) in \(w\)

d. i. Decision Problem1: In Amsterdam, questioner is surveying availability of Italian press for a travel website (actions corresponding to writing down different sets of places where Italian press is sold)
   ii. Decision Problem2: In Amsterdam, questioner wants to read news in Italian by buying the newspaper at the station (Action1), or at the palace (Action2)

For the exhaustivity dimension, the Decision Problem dependent denotation in (44c) results in vagueness being resolved depending on the goals of the interlocutors in the situation. Decision problem in (44d-i) resolves the vagueness by pointing to the mention-all answer, while the one in (44d-ii) supports the mention-some interpretation. In the same way as it does for plurals, this framework results in
mention-all question interpretations when in each world, the optimally-relevant answers are singleton sets. Exactly when several answers are equally optimally-relevant, the REL will yield a mention-some interpretation for the questions.

5.2 Overt distributivity and maximality operators

Not all sentences with definite plurals are vague. Operators like all or each force maximal (and, in the case of each, fully distributive) interpretations in sentences with definite plurals (45), compare with (46).

(45) a. All the boys surrounded the castle.
   b. The boys built a raft each.

(46) a. The boys surrounded the castle.
   b. The boys built a raft.

How do these lexical items achieve this effect? A collective sentence like (45a) demonstrates that all cannot be working to eliminate non-maximal readings from the pool of propositions that REL considers. That is because in our semantics, based on Landman’s groups, a collective predicate applies to a group and thus cannot be forced to be maximal. Thus, maximality does not arise from the same pool of readings available for sentences without all (46a), but is an additional condition.

Moreover, this maximality condition cannot apply after the eliminative calculation conducted by REL, since all is capable of inducing maximality in situations where maximal readings would be eliminated by REL. For instance, in (47) below, REL would eliminate the maximal distributive proposition in (47b) as over-informative, since even one or two windows would necessitate a return to the house (cf. 32).

(47) a. All the windows are open.
   b. **Actual interpretation:** $w_1 \sqcup w_2 \sqcup \cdots \sqcup w_{10}$ is open
   c. **Decision Problem:** Before a thunderstorm, the interlocutors want to decide if they need to return to the house (which has 10 windows) to make sure it is rain-proof.

Brisson (1998) proposes that all ensures that the packaging of entities within the plural NP is maximal by restricting the value of the Cover variable on which the distributivity operator depends. Specifically, all restricts this variable to range only over well-fitting covers, in which the cells of the cover are all either subsets of the plural NP denotation or subsets of the complement of that denotation. Since ill-fitting covers (those that allow salient exceptions) are the only way to achieve non-maximality in Brisson’s framework, the good fit requirement precludes non-maximal interpretations. The approach is similar to the way in which pronominal
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features, such as gender, restrict the value of the variable denoted by the pronoun to range over individuals of that gender.

The drawbacks of Brisson’s salient cover-based framework in accounting for non-maximal interpretations of definite plurals were discussed in section 3.3. Independently, Brisson’s proposal for all uses its maximality-inducing effect to explain its inability to license collective readings with certain types of predicates, such as be numerous. As pointed out in Champollion (2010), this connection is undesirable, because non-maximal operators such as most, many, few also share this inability (48).

(48) a. [anomalous] All the boys are numerous.  
   b. [anomalous] Most of the boys are numerous.  
   c. [anomalous] Few of the boys are numerous.

In contrast to Brisson’s pragmatic approach, Winter (1998), Winter (2002) proposes that all works in the semantics to change the available interpretations. In Winter (2002)’s framework NP and VP denotations can denote sets of atomic entities, or sets of sets of entities. He defines all as a composite determiner dfit(every), where every has the usual denotation (a relation between sets of atomic entities), while dfit operation is defined as in (49).

(49) Let \( D \) be a relation between sets of elements in a domain \( E \) of atoms. The operator \( \text{dfit} \) (determiner fitting) maps \( D \) to a relation \( \text{dfit}(D) \) between sets of sets of atoms in \( p(E) \), which is defined as follows:  
For any two sets \( A, B \in p(E) \), the relation \( (\text{dfit}(D))(A, B) \) holds iff the relation \( D(\bigcup A, \bigcup (A \cap B)) \) holds.

Translating this to the notation used here (with groups and sums), and making this specific to all, with some modifications we get (50). Without REL, this would produce the denotation in (51) for (45a).

(50) \[ \text{all} = \text{dfit}(\text{every}) = \lambda P.\lambda Q.\lambda e.\forall x[P(x) \rightarrow \exists y[Q(e) \& \theta(x)]\] 
   \( = \lambda P.\lambda Q.\lambda e.\forall x[P(x) \rightarrow \exists y[Q(e) \& \theta(x) = y \& (x \leq y) \& \forall z[(z \leq y) \rightarrow P(z)]]] \]

(51) \[ [\text{all the boys surrounded the castle}] = \]  
\[ = \text{Every}(\sigma x. * \text{boy}(x), \bigcup (\sigma x. * \text{boy}(x) \cap \sigma x. \text{surrounded.caste}(x))) \]  
\[ = \text{Every boy participated in a set of boys that surrounded the castle} \]

How would this proposal for all interact with the \( O_{Rel} \) operator? In the spirit of Winter’s approach, perhaps it resolves the vagueness of the NP, so that the pragmatic relevance calculation has nothing further to do. In that case, as in the case without all, \( O_{Rel} \) will operate on the utterance to resolve any vagueness. However, the NP containing all is no longer vague with respect to maximality. As a result, while REL will still produce a set of propositions in the end (because of the type mismatch,
combining via function combination, as before), in non-vague utterances this will be
a singleton set. For a sentence like (45a), it will be a set containing the proposition
that all the boys surrounded the castle.

As appealing as this simple proposal is, there are two reasons to think that it
wouldn’t do. First, note that while the lexical entry in (50) derives the effect of all on
the maximality of the resulting interpretation, the resulting interpretation is still vague
with respect to distributivity, as the predicted readings in (52) indicate\(^{10}\). However,
there is no obvious way for REL to apply and resolve this residual vagueness, since
the denotation of all the boys is no longer a sum or group individual, but rather a
quantificational expression.

(52) All the boys built a raft.
   a. Three boys’ teams and two girls’ teams compete in building a team raft:
      intermediate distributive reading
   b. Individual boys and girls compete in building individual rafts: fully
distributive reading

Second, Winter (2002)’s proposal for all is not sufficient. As Champollion (2010)
points out, all is incompatible with cumulative readings in sentences like (53) below,
while still allowing such readings in sentences with bare plurals, such as (54).

(53) All the professors graded 30 papers.
   a. Cumulative reading (unavailable): every professor participated in a set
      of professors who graded 30 papers (and each of the 30 papers was
      graded by a professor).
   b. Actual reading: Every professor graded 30 papers, so that if there are
      three professors, 90 papers were graded.

(54) All the professors graded papers.
   "Cumulative" reading: Every professor participated in a set of professors
   who graded papers.

The lexical entry in (50) makes the incorrect prediction for (53) because it
does, in fact, leave the utterance vague with respect to distributivity. To resolve
this vagueness and derive the contrast between (53) and (54), Champollion (2010)

\(^{10}\) The sentence in (52) does not have a collective reading (i), for a mysterious reason. A variant of this
sentence with an adverbial all (ii) does have this reading. Neither Brisson (1998) nor other authors, to
my knowledge, point out this difference in the interpretation of prenominal and adverbial all. Special
thanks go to Charles Searing (p.c.) for pointing it out.

i. A boys’ team and a girls’ team compete in building a team raft: collective reading.

ii. The boys all built a raft.
proposes that *all* carries a presupposition that he terms a requirement of Stratified Reference, defined for *all* in (55) below.

\begin{equation}
SR_{\theta,\text{Atom}}(Q) = \text{def} \forall e [Q(e) \rightarrow e \in ^{*} \lambda e'(Q(e') \& \text{Atom}(\theta(e')))]
\end{equation}

(A predicate \( P \) has stratified reference with respect to a theta-role \( \theta \) and a (distributivity) threshold \( \text{Atom} \) if and only if there is a way of dividing every eventuality in its denotation exhaustively into parts (“strata”) which are each in \( P \) and whose theta-roles \( \theta \) satisfy the condition of being atomic.)

In Champollion’s system, the presupposition of Stratified Reference is satisfied whenever the sentence contains a distributivity operator (which may be dependent on a decision problem). He leaves the maximality-inducing effect of *all* unexplained.

I propose, unifying the insights of the proposals in Brisson (1998), Winter (2002), and Champollion (2010), that the effect of *all* on its attached NP introduces two requirements. One requirement, following Champollion (2010), demands Stratified Reference (55, 56). The other requirement, demanding maximality, has a dual effect on the NP with which *all* combines. First, it introduces a maximality presupposition, implemented along the lines of Winter (2002) (50, 50’, 56). Second, an NP carrying this presupposition is no longer vague with respect to maximality, and thus acquires a special feature MAX (56), intervening in the pragmatic calculation along the lines of Brisson (1998).

\begin{equation}
\text{MAX}([NP]) = \lambda Q. \lambda e: \text{MAX}([NP](Q)(e) \&SR_{\theta,\text{Atom}}(Q), [Q(e) \& ^{*} \theta(e)] = [NP])
\end{equation}

The maximality-inducing item *all* marks the NP in which it appears with the MAX feature, which is passed up and thus communicated to the \( O_{\text{Rel}} \) (57). When \( O_{\text{Rel}} \) is coindexed with an NP that contains *all*, the REL operator considers only propositions that are compatible with maximal interpretations (57). The REL operator, applying to a vague utterance containing a vague predicate and the *all*-NP compares the relevance of those NP-made (e.g., boy-made) entities that comprise all the boys; that is, those that are a good fit for the NP (Brisson 1998).

\begin{enumerate}
\item The structure for agentive NP with *all*:
\end{enumerate}
b. The relevance operator for maximal NPs:

**Definition:** \( REL(DeP)(X_{n\theta-MAX})(w) = \)
\[ \{ g : \uplus\{ \downarrow d : d \in AT(\downarrow g) \} = \downarrow [NP_{\theta_n}](w) \}
\]

Paraphrase: \( REL \) is a function that takes the decision problem, a proposition containing a predicate and its argument NP (marked as maximal) denotations, and a world, and outputs a set of individuals \( g \), which could be groups or sums.

The first conjunct (double-underlined) assures that when you boil \( g \) down to singularities, all those singular individuals are atoms that make up the entire NP denotation (e.g., 'all the children').

The second conjunct assures that no other such NP-comprising (e.g., comprising all of the atomic children in the NP) thing \( h \), is more relevant than \( g \) – as the single-underlined portion states, there is no \( h \) such that it is better to learn the proposition \( [X] \) about \( h \) fulfilling the predicate, than to learn the proposition \( [X] \) about \( g \) fulfilling the predicate.

c. \( O_{Rel}^{1\theta-MAX} = \lambda w.\lambda \phi.\lambda x.\lambda e.\phi = [X_{1\theta-MAX}] \)
\( x \in \text{spelling out the contents of the combined proposition} \)
\[ = \lambda w.\lambda \phi.\lambda x.\lambda e.\phi = [X_{1\theta-MAX}] \]
\( x \in \text{REL}(DeP)(X_{1\theta-MAX})(w)\)&\([X][x/[NP_{\theta-MAX}]]\) =

For instance, the presence of *all* in the agentive NP produces the following meaning calculation for the sentences with a collective reading (58), and those with some level of distributivity (59), assuming there are a total of three boys. First, consider the utterance in (45a), in which *all* enforces maximality in a plural agent of
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a collective predicate.

(45a) All the boys surrounded the castle.

(58) a. \[[IP_{Ag-Max}] = \lambda w \lambda e : MAX(\uparrow \sigma x * boy(x, w))(\lambda e * surround(e, w))(e) &
\& SR_{Ag,Atom}(\lambda e [*surround(e) & Theme(e) = \lambda x castle(x)])].

\*surround(e, w) & Theme(e) = \lambda x castle(x) & Ag(e) = \uparrow \sigma x * boy(x, w)

b. The pool of propositions considered by REL:

\*surround-castle(b1 ⊔ b2 ⊔ b3) distributive reading - no boy can surround the castle by himself

\*surround-castle(\uparrow (b1 ⊔ b2 ⊔ b3)) collective reading

c. \lambda w: \exists p[p \in \{\lambda w: [*surround(e, w) & Theme(e) = \lambda x castle(x)] &
\& Ag(e) = \uparrow (b1 ⊔ b2 ⊔ b3)\}] & p = 1

Presupposition:

MAX(\uparrow \sigma x * boy(x, w))(\lambda e * surround(e, w))(e) &
\& SR_{Ag,Atom}(\lambda e [*surround(e) & Theme(e) = \lambda x castle(x)])

(\text{Every boy participated in the plurality of boys that is the agent of the event of surrounding the castle and every event in which the castle was surrounded consists of one or more surrounding events whose agents are atomic and whose theme is the castle.})

The MAX feature ensures that only pluralities involving all three boys are considered by the REL (58b), the corresponding maximality presupposition enforces the participation of all three boys in the actual event. At the same time, the stratified reference requirement is satisfied, since the event of surrounding the castle consists of a single surrounding event whose agent is the impure atom (underlyingly consisting of the three boys) and whose theme is the castle.

Now, consider the meaning calculation of the vague sentence in (59), which is maximal, but compatible with a full or intermediate distributive reading. In a situation where two teams of boys (two boys each) compete against three teams of girls in building a team raft, the sentence is interpreted as intermediate distributive.

(52) All the boys built a raft.

(59) a. \[[IP_{Ag-Max}] = \lambda w \lambda e : MAX(\uparrow \sigma x * boy(x, w))(\lambda e * build(e, w))(e) &
\& SR_{Ag,Atom}(\lambda e [*surround(e) & \exists x[raft(x) & Theme(e) = x]])].

\*build(e, w) & \exists x[raft(x) & Theme(e) = x] & Ag(e) = \uparrow \sigma x * boy(x, w)

b. The pool of propositions considered by REL\(^{11}\):

11 I am pretending here that this sentence has a collective reading. It doesn’t - in a situation requiring a collective maximal interpretation, only the adverbial \textit{all} is possible:

i. The boys all built a rat.
*build-raft\((b_1 \sqcup b_2 \sqcup b_3 \sqcup b_4)\)

distributive reading - overinformative

*build-raft\((\uparrow (b_1 \sqcup b_2 \sqcup b_3 \sqcup b_4))\)

collective reading - underinformative

*build-raft\((\uparrow (b_1 \sqcup b_2) \sqcup \uparrow (b_3 \sqcup b_4))\)

intermediate distributive - resolves the DeP
c.

\(\lambda w. \exists p[p \in \{\lambda w.[*build(e,w)\&\exists x[raft(x)\&*Theme(e) = x]\&*Ag(e) = (\uparrow (b_1 \sqcup b_2) \sqcup \uparrow (b_3 \sqcup b_4))]\} \& p = 1]\)

Presupposition:

\([MAX(\uparrow \sigma x * boy(x,w))(\lambda e. *build(e,w))(e)\& \&SR_{Ag,Atom}(\lambda e[*build(e)\&\exists x[raft(x)\&*Theme(e) = x]])]\)

(Every boy participated in the plurality of boys that is an agent of an event building a raft and every event in which a raft was built consists of one or more building events whose agents are atomic and whose theme is a raft.)

The pool of propositions includes several possibilities for distributivity, only one of which is selected as optimal by REL.

The stratified reference requirement rules out the unavailable cumulative reading in (53) because under that reading the requirement is not satisfied (60) (Champollion 2010). Stratified reference requires that every eventuality in the denotation of graded 30 papers can be divided into events with atomic agents, which are still events of grading 30 papers. The cumulative reading violates this last part - under this reading, each of the professors is the agent of a paper-grading event that add up to grading 30 papers, but a sub-event of which an individual professor is an agent is not an event of grading 30 papers. For instance, each professor could grade 10 papers. In a situation where there are three professors, the cumulative reading would be represented as in (60).

(53) All the professors graded thirty papers.

(60) \(\lambda w. \exists p[p \in \{\lambda w.[*grade(e,w)\&\exists x[paper(x)\&*Theme(e) = x]\&*Ag(e) = (\uparrow (p_1 \sqcup p_2 \sqcup p_3))]\} \& p = 1]\)

Presupposition:

\([MAX(\uparrow \sigma x * professor(x,w))(\lambda e. *grade(e,w))(e)\& \&SR_{Ag,Atom}(\lambda e[*grade(e)\&\exists x[paper(x)\&*Theme(e) = x]\&|x| = 30]])]\)

(Every professor participated in the plurality of professors that is the agent of the event of grading 30 papers and every event in which 30 papers were graded consists of one or more grading events whose agents are atomic and whose themes are sums of 30 papers.)
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In contrast, the distributive reading, in which each professor ends up grading 30 papers (for a total of 90 overall when there are three professors), satisfies stratified reference: each sub-event has an atomic professor-agent and is an event of grading 30 papers.

Finally, the contrast with (54) is also explained by the stratified reference presupposition: each individual (atomic) professor is an agent of a sub-event which still satisfies the description graded papers (61).

(54) All the professors graded papers.

(61) $\lambda w. \exists p [ p \in \{ \lambda w. [*\text{grade}(e,w) \& \exists x [ *\text{paper}(x) \& \text{Theme}(e) = x ] \& \exists \text{Ag}(e) = (\uparrow (p1 \sqcup p2 \sqcup p3))\} \& p = 1]$

Presupposition:
$\text{MAX}(\uparrow \sigma x [ *\text{professor}(x,w)](\lambda e. *\text{grade}(e,w))(e) \&$
$\text{SR}_{\text{Ag}, \text{Atom}}(\lambda e [ *\text{grade}(e) \& \exists x [ *\text{paper}(x) \& \text{Theme}(e) = x ]])$

(Every professor participated in the plurality of professors that is the agent of the event of grading papers and every event in which papers were graded consists of one or more grading events whose agents are atomic and whose themes are sums of papers.)

While it is not the goal of this paper to provide a detailed treatment of all or other lexical maximality and distributivity operators, my proposal demonstrates that a pragmatically-driven account of vagueness in definite plurals is compatible with an analysis of such operators.

5.3 Some further issues


(62) a. For the most part, Al knows about which kids are drunk.
     b. For the most part, Al hates the kids on his block.
     c. For the most part, Al knows where the kids are hiding.

Quantification over parts can apply to wh-phrase denotations (62a) as well as to definite plurals (62b). Sometimes, it can apply to either, depending on the context (Williams 2000) (62c). A framework using similar mechanisms for interpreting definite plurals and wh-phrases is a prerequisite for addressing this data.
References


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