

Math 21b Review Sheet for the Midterm Exam

Here is what **exam problems** may look like.

Items marked by [Q4] may be incorporated into the quiz (March 9).

- For an explicitly given function from \mathbb{R}^n to \mathbb{R}^m , find its domain (that is, the set of points where the function is defined)
- For a point in \mathbb{R}^n moving according to $\mathbf{x} = f(t)$, $a \leq t \leq b$, compute: velocity, speed, acceleration, the length of the trajectory (= the arclength of the curve parametrized by f)
- Reconstruct f as above from its first or second derivatives and some initial conditions (see Problems 7–14 in §4.1F)
- For a given function, decide at which points of its domain it is continuous/differentiable (see Problems 26–31 in §5.1 and 17–20 in §5.3)
- For a given set, find its: limit points, interior points, boundary points; decide whether the set is open or closed or neither (see Problems 2–12 in §5.1)
- For a given differentiable function f from \mathbb{R}^n to \mathbb{R}^m , find its: partial derivatives (first order and higher order), derivative matrix, tangent approximation [Q4]
- For a vector \mathbf{v} and f as above, find: the derivative $\frac{\partial f}{\partial \mathbf{v}}$ of f with respect to \mathbf{v} , the directional derivative in the direction of \mathbf{v} (by definition equal to $\frac{1}{\|\mathbf{v}\|} \frac{\partial f}{\partial \mathbf{v}}$) [Q4]
- Describe tangent lines/planes to curves/surfaces [Q4]; here you simply need to remember the main principle as follows:

THE TANGENT LINE/PLANE TO A _____ OF A FUNCTION AT A GIVEN POINT
IS THE _____ OF ITS TANGENT APPROXIMATION AT THIS POINT

Here _____ may stand for “image” (parametrized curves/surfaces) or “level set” or “graph”.

- Use tangent approximation to find approximate values of functions
- Compute the derivative of the composition of two functions using the chain rule (see §6.1B and §6.2A)
- For a function from \mathbb{R}^n to \mathbb{R} and a point \mathbf{x}_0 , find a vector pointing in the direction of maximum increase of f at \mathbf{x}_0 , and a vector normal to the level set of f containing \mathbf{x}_0 (both are given by ∇f); use it to describe the tangent line/plane to the level set (see §6.1AC).

The complete **list of sections** on which the quiz is based is:

4.1ABCDF, 4.2AB, 4.3ABC, 4.4AB, 5.1ABC, 5.2AB, 5.3AB, 5.4ABC, 6.1ABC, 6.2A.

There will be no problems explicitly asking you to **sketch** a curve or a surface, yet in some of the problems a sketch may happen to be helpful for the solution.