Math 23b	**Homework 8**	**Spring, 2008**

Due Wednesday, March 26

From the textbook, chapter 4:

4.43, 4.45, 4.49
Optional (for extra credit): 4.51

Also do the following two problems:
(∗) Let $A_1 \supseteq A_2 \supseteq \ldots$ be an infinite family of sets such that A_1 is finite, and for each $k \in \mathbf{N}$, $A_{k+1} \subseteq A_k$. Prove that this family eventually stabilizes, that is, there exists $n \in \mathbf{N}$ such that for all $k \geq n$, $A_k = A_n$. [Hint: use induction on the size of A_1.]

(∗∗) Prove that every infinite subset A of $\mathbf{N}$ is countable, that is, that there exists a bijection $f : \mathbf{N} \to A$. [Hint: think of bijections constructed in class for $A = \{\text{even numbers}\}$ or $\{\text{powers of } 2\}$, and try to come up with a general way of constructing a bijection. The Well-Ordering Principle (Proposition 3.30) can be helpful too.]