Physics 162a, Fall 2000 – Problem Set 1

Due in class Wednesday, Sept. 16

1. Baym, Chapter 1, problem 1.

2. Baym, Chapter 1, problem 9.

3. Baym, Chapter 1, problem 11.

4. Consider the following three elements from the *real* vector space of real 2×2 matrices:

$$|1\rangle = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad ; \quad |2\rangle = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \quad ; \quad |3\rangle = \begin{pmatrix} -2 & -1 \\ 0 & -2 \end{pmatrix} \tag{1}$$

Figure out whether or not they are linearly independent (and prove your claim).

5. Show that the following vectors in $\mathbb{C}^3$

$$|I\rangle = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} \quad ; \quad |II\rangle = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \quad ; \quad |III\rangle = \begin{pmatrix} 0 \\ 2 \\ 5 \end{pmatrix} \tag{2}$$

are linearly independent. Using the Gram-Schmidt procedure, use these to construct an orthonormal basis.

6. Imagine the world is a one-dimensional lattice: that is, a set of points at $x = 0, \epsilon, 2\epsilon, \ldots, N\epsilon$.

   (a) Show that the space of complex-valued functions on the lattice, with periodic boundary conditions $f(0) = f(N\epsilon)$, is a complex vector space. Show that the inner product

$$\langle f|g\rangle = \epsilon \sum_{n=0}^{N-1} f^*(n\epsilon)g(n\epsilon) \tag{3}$$

   defines an inner product space with an adjoint, where f^* is the complex conjugte of f. (The factor of ϵ in this sum is necessary for the limit $\epsilon \to 0$, $N \to \infty$, $N\epsilon \to L < \infty$ to be well defined.)

(b) Write a function $X_k(x)$ (for $0 \leq k \leq N-1$) such that $\langle X_k | f \rangle = f(k\epsilon)$. Compute $\langle X_k | X_{k'} \rangle$. Is this an ortho*normal* basis? If not, does the inner product have a limit as $\epsilon \to 0$?

(c) You can decompose functions on this lattice in a Fourier series. Show that

$$\tilde{f}_k(x) = A_k \exp\left(\frac{2\pi i k x}{N\epsilon}\right) \tag{4}$$

for $0 \leq k \leq N-1$ is a complete orthonormal basis for the correct A_k, and find that A_k. Write this orthonormal basis in terms of the basis $|X_m\rangle$.