Physics 162a, Fall 2008 – Problem Set 3
Due Monday October 6

1. Fun with the polarization of spin-1/2 systems. For a 3d vector $\vec{n}$ with unit norm, the operator measuring spin along $\vec{n}$ is

$$S(\vec{n}) = \frac{\hbar}{2} \vec{\sigma} \cdot \vec{n} = \frac{\hbar}{2} (n_x \sigma_x + n_y \sigma_y + n_z \sigma_z),$$

where $\sigma^i$ are the Pauli matrices. Imagine I have a beam of spin-1/2 particles travelling with their spins aligned along the $z$ direction. Now place a polarizer in the way which selects particles whose spin points along an angle $\theta$ away from the $z$ axis. Compute the probability that a given particle will make it through the polarizer, as a function of $\theta$, and compute its final state in the $z$ basis.

2. Construct the unitary operator which transforms the basis of eigenvectors of $\sigma_z$ to the basis of eigenvectors of $\sigma_x$.

3. Consider a 3-dimensional Hilbert space and the observable:

$$\mathcal{O} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Find the normalized (unit norm) eigenvectors and eigenvalues. Is there a degeneracy?

4. Consider the observables

$$A = \begin{pmatrix} a & 0 & 0 \\ 0 & -a & 0 \\ 0 & 0 & -a \end{pmatrix}; \quad B = \begin{pmatrix} b & 0 & 0 \\ 0 & 0 & -ib \\ 0 & ib & 0 \end{pmatrix}$$

(a) A clearly has a degenerate spectrum. Does B?
(b) Show that A and B commute.
(c) Find a basis which diagonalizes both A and B, and the eigenvalues of A and B for each simultaneous eigenvector. Does specifying the eigenvalues of both A and B specify the state completely?