Physics 162b, Winter/Spring 2009 – Problem Set 5.2

Due in class Tuesday, April 28

1. Two nonidentical spin-1 particles can form states with total angular momentum $j = 0, 1, 2$. What are the restrictions for identical particles?

2. Baym, Problem 19.1

3. Baym, Problem 19.3 (will require reading the part in sec 19 on pair correlation functions).

4. A homogenous system of spin-0 particles in a cubic box of size L, as $L \to \infty$, is described in the second-quantized description by the Hamiltonian

$$\begin{aligned} H &= \frac{\hbar^2}{2m}\int d^3x \vec{\nabla}\hat{\phi}^+(x)\cdot\vec{\nabla}\hat{\phi}^- \\ &\quad +\frac{1}{2}\int\int d^3x d^3x' \hat{\phi}^+(x)\hat{\phi}^+(x')V(\vec{x}-\vec{x}')\hat{\phi}^-(x)'\hat{\phi}^-(x) \end{aligned} \quad (1)$$

We will treat V as a perturbation.

(a) Show that the expectation value of the interacting Hamiltonian, in the ground state of N bosons for the noninteracting system, is

$$\frac{E^{(1)}}{N} = \frac{(N-1)\tilde{V}(0)}{2L^3} \quad (2)$$

where

$$V(\vec{x}) = \int d^3k \frac{1}{(2\pi)^3}\tilde{V}(k)e^{i\vec{k}\cdot\vec{x}} \quad (3)$$

The factor of L^3 is a bit tricksy. It will show up via the normalization of the states. If you express the ground state in terms of some momentum basis, the norm of this states will have a bunch of delta functions in momentum space with argument 0. These are $\delta^{(3)}(k-k) = \delta^{(3)}(k)$ and you can set that delta function equal to L^3 by dimensional analysis. Next, since the state itself has norm proportional to L^{3N} rather than to 1, the expression for the energy shift to first order in perturbation theory is:

$$\delta E^{(1)} = \frac{\langle\psi|H|\psi\rangle}{\langle\psi|\psi\rangle} \quad (4)$$

which is dimensionally correct. Alternatively, you can re-develop the formalism working at finite L always. Either of these are acceptable, but either way you should explain your reasoning.

(b) Show that the second-order contribution (in V) to the ground-state energy is:

$$\frac{E^{(2)}}{N} = -\frac{N-1}{L^3}\int d^3q \frac{|\tilde{V}(q)|^2}{(\hbar^2 q^2)/m} \approx -\frac{n}{2}\int d^3q \frac{|\tilde{V}(q)|^2}{(\hbar^2 q^2)/m} \quad (5)$$

where $n = N/L^3$ is the particle density.

(c) If $V|(x|) < 0$ for all $\vec{x}$, and if $\int d^3x |V(x)|^2 < \infty$, show that the system will collapse (Hint: start from $E^{(1)}/N$ as a function of the density).