

# Physics 162b – Quantum Mechanics, winter 2009

## Problem Set 3

Due at the beginning of class on Tuesday, March 24.

1. Derive an expression for the density of free particle states  $\rho dE d\Omega$  in *two* dimensions. Your answer should be written as a function of  $k$  or  $E$ , times  $dE d\phi$ , where  $\phi$  is the polar angle of the momentum in 2d.
2. (From Messiah). In  $\beta$  decay, an atomic nucleus emits an electron with velocity usually close to  $c$ , and its charge goes from  $Ze$  to  $(Z+1)e$ . Show that the effect of this transition on the other electrons can be treated in the sudden approximation (I sketched this in class). Verify that this method applies to the decay of tritium ( $H^3$ : 1 proton and 2 neutrons) into  $He^3$  (2 protons and 1 neutron), where the kinetic energy of the emitted electron is only 16 keV. If the electron orbiting the nucleus in the triton atom (this is \*not\* the same as the electron emitted in the decay to  $He^3$ !) is in the ground state, compute the probability that the  $He^3$  ion found after the decay will be in the  $1S$  state, in the  $2S$  state, and in any  $\ell \neq 0$  state.
3. Consider a 1d simple harmonic oscillator with a time-dependent frequency,  $\omega(t)^2 = \omega_0^2 + \frac{1}{2}\delta\omega^2 \text{sech}^2\left(\frac{t}{\tau}\right)$ , and the corresponding time-dependent Hamiltonian  $H(t)$ 
  - (a) Under what conditions is the adiabatic approximation valid?
  - (b) Do not assume the adiabatic approximation holds. Under what circumstances is time-dependent perturbation theory valid? Compute the probability of making a transition from the ground state at  $t = -\infty$  to any excited state of  $H(\infty)$ , as  $t \rightarrow \infty$ , to first order in time-dependent perturbation theory.