1. Baym, 9.1
2. Baym, 9.2
3. Baym, 10.1

4. (Merzbacher, 20.4) Using the Born approximation, express the differential cross section for the scattering of an electron off of a spherically symmetric charge distribution \( \rho(r) \), as the product of the Rutherford scattering cross section (this is the classical cross section or equivalently the cross section for a Yukawa potential \( V = -q^2 e^{-\mu r} / r \) in the limit \( \mu \to 0 \)) and the square of a form factor \( F \). Obtain an expression for the form factor, and evaluate it as a function of the momentum transfer (the difference between the initial and final momentum vectors) for
   (a) a uniform charge distribution of radius \( R \), and
   (b) a Gaussian charge distribution with root mean square radius \( R \).

5. (Sakurai, problem 7.2) Prove that

\[
\sigma_{\text{tot}} = \frac{m^2}{\pi \hbar^4} \int d^3 x \int d^3 x' V(x) V(x') \frac{\sin^2(k |\vec{x} - \vec{x}'|)}{k^2 |\vec{x} - \vec{x}'|^2} \tag{1}
\]

in each of the following ways:
   (a) By integrating the differential cross section computed using the first-order Born approximation.
   (b) By applying the optical theorem to the forward-scattering cross section in the second-order Born approximation.

6. (Sakurai problem 7.9) Consider scattering by a repulsive \( \delta \)-shell potential:

\[
V(r) = \frac{\hbar^2 \gamma}{2m} \delta(r - R); \quad \gamma > 0 \tag{2}
\]
(a) Set up an equation that determines the S-wave phase shift $\delta_0$ as a function of $k$.

(b) Assume now that $\gamma$ is very large,

$$\gamma \gg \frac{1}{R}, k$$

(3)

Show that if $\tan kR$ is *not* close to zero, the S-wave phase shift resembles the hard-sphere result (see Sakurai). Show also that for $\tan kR$ close to but not exactly zero, resonance behavior is possible: that is, $\cot \delta_0$ goes through zero from the positive side as $k$ increases. Determine approximately the positions of the resonances keeping terms of order $1/\gamma$; compare them with the bound-state energies for a particle confined *inside* a spherical wall of the same radius,

$$V = 0, \quad r < R; \quad V = \infty, \quad r > R$$

(4)

Also obtain an approximate expression for the resonance width $\Gamma$ defined by

$$\Gamma = \frac{-2}{|d(\cot \delta_0)/dE|_{E=E_r}}$$

(5)

and notice in particular that the resonances become extremely sharp as $\gamma$ becomes large.