Physics 162b – Quantum Mechanics, winter 2009

Problem Set 4.1 – scattering theory

Due at the beginning of class on Tuesday, April 28.

1. Baym, 9.1
2. Baym, 9.2
3. Baym, 10.1
4. (Merzbacher, 20.4) Using the Born approximation, express the differential cross section for the scattering of an electron off of a spherically symmetric charge distribution $\rho(r)$, as the product of the Rutherford scattering cross section (this is the classical cross section or equivalently the cross section for a Yukawa potential $V = -q^2 e^{-\mu r}/r$ in the limit $\mu \to 0$) and the square of a *form factor* F. Obtain an expression for the form factor, and evaluate it as a function of the momentum transfer (the difference between the initial and final momentum vectors) for
   (a) a uniform charge distribution of radius R, and
   (b) a Gaussian charge distribution with root mena square radius R.
5. (Sakurai, problem 7.2) Prove that
$$\sigma_{tot} = \frac{m^2}{\pi\hbar^4} \int d^3x \int d^3x' V(x) V(x') \frac{\sin^2(k|\vec{x} - \vec{x}'|)}{k^2 |\vec{x} - \vec{x}'|^2} \tag{1}$$
   in each of the following ways:
   (a) By integrating the differential cross section computed using the first-order Born approximation.
   (b) By applying the optical theorem to the forward-scattering cross section in the second-order Born approximation.
6. (Sakurai problem 7.9) Consider scattering by a repulsive δ-shell potential:
$$V(r) = \frac{\hbar^2 \gamma}{2m} \delta(r - R) \; ; \qquad \gamma > 0 \tag{2}$$

(a) Set up an equation that determines the S-wave phase shift δ_0 as a function of k.

(b) Assume now that γ is very large,

$$\gamma >> \frac{1}{R},\ k \tag{3}$$

Show that if $\tan kR$ is *not* close to zero, the S-wave phase shift resembles the hard-sphere result (see Sakurai). Show also that for $\tan kR$ close to but not exactly zero, resonance behavior is possible: that is, $\cot \delta_0$ goes through zero from the positive side as k increases. Determine approximately the positions of the resonances keeping terms of order $1/\gamma$; compare them with the bound-state energies for a particle confined *inside* a spherical wall of the same radius,

$$V = 0\ , \quad r < R\ ; \qquad V = \infty, \quad r > R \tag{4}$$

Also opbtain an approximate expression for the resonance width Γ defined by

$$\Gamma = \frac{-2}{[d(\cot \delta_0)/dE]|_{E=E_r}} \tag{5}$$

and notice in particular that the resonances become extremely sharp as γ becomes large.