Hebbian learning and Hopfield networks

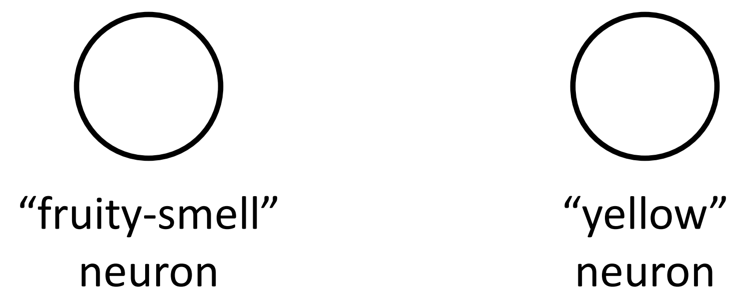
Pietro Berkes, Brandeis University

“Classical” models of learning

- Characterized by deterministic update and learning rules
- The models have a number of parameters that are adjusted such that the model performs a certain function
- Examples: neural networks, support vector machines, PCA, …
- Issues: cannot cope with uncertainty

Hebb's rule

- Most of learning in the brain is unsupervised
- Brilliant idea by Hebb (1949):
  cells that fire together, wire together



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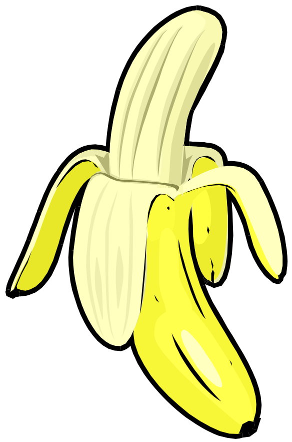

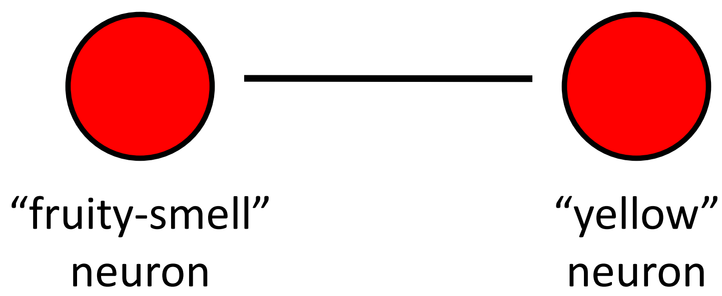

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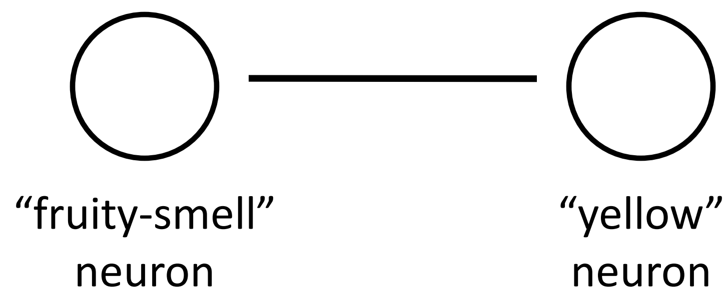

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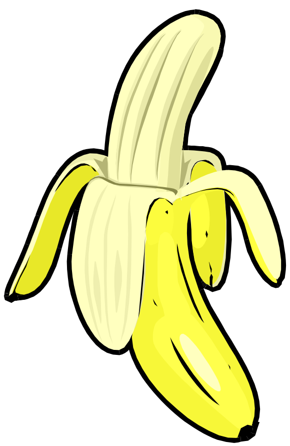

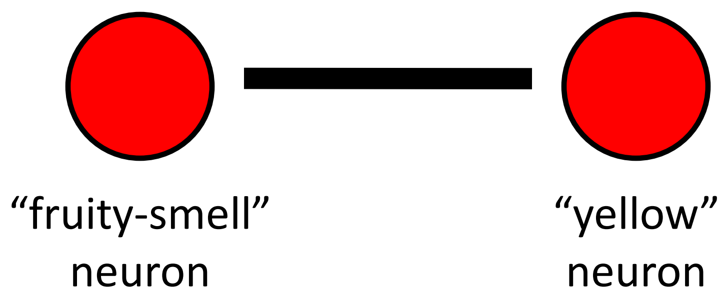

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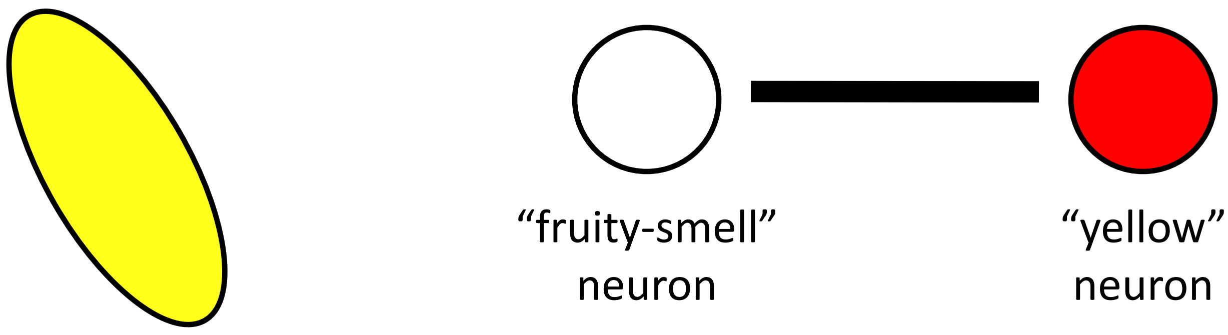

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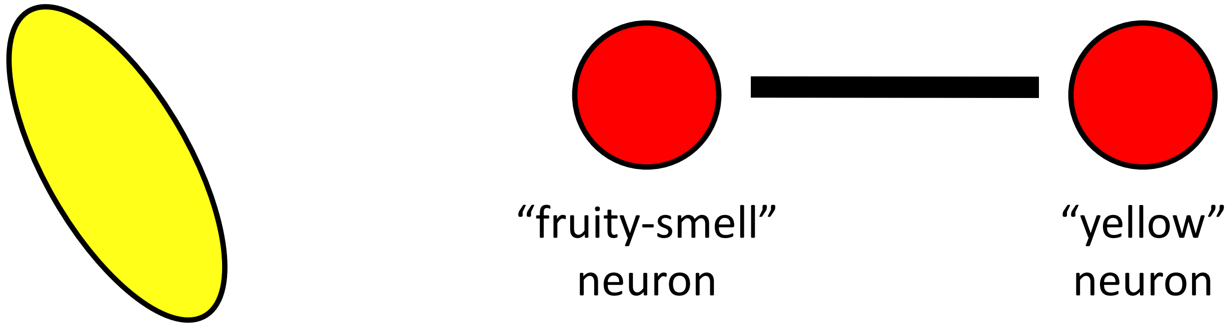


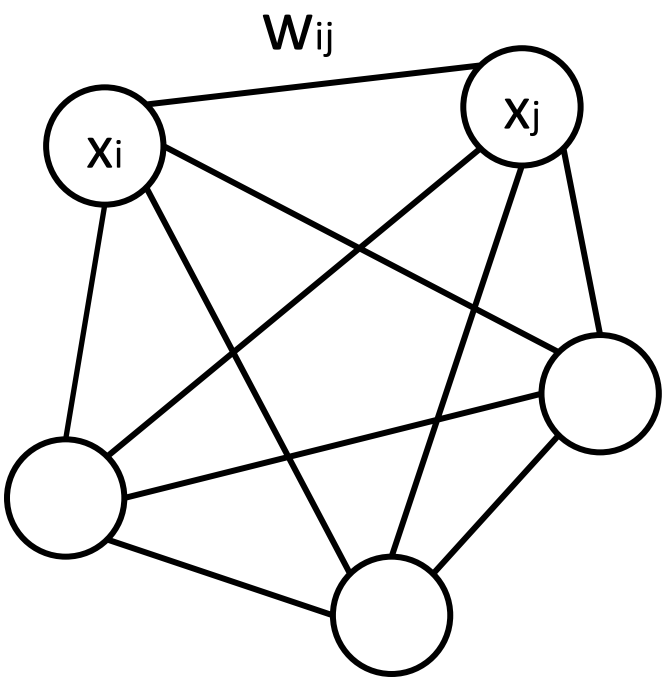
Hopfield network

- Hebb’s ideas where formalized much later: Hopfield network (1982)
- Most direct implementation of Hebb’s ideas:

$$\Delta w_{ij} = x_i x_j$$

- Note: local learning rule, no teacher

Architecture



- x_i = +1 or -1
  (binary neural activity)

- $w_{ii} = 0$ (no self-connections)
- $w_{ij} = w_{ij}$ (symmetric, bidirectional connections)

Learning rule

- Set neurons x_i to desired pattern
- Update weights as $\Delta w_{ij} = x_i x_j$

- i.e., if we have N patterns $x_i^{(n)}$
  the final weights will be

$$w_{ij} = \sum_{n=1}^{N} x_i^{(n)} x_j^{(n)}$$

Activity rule

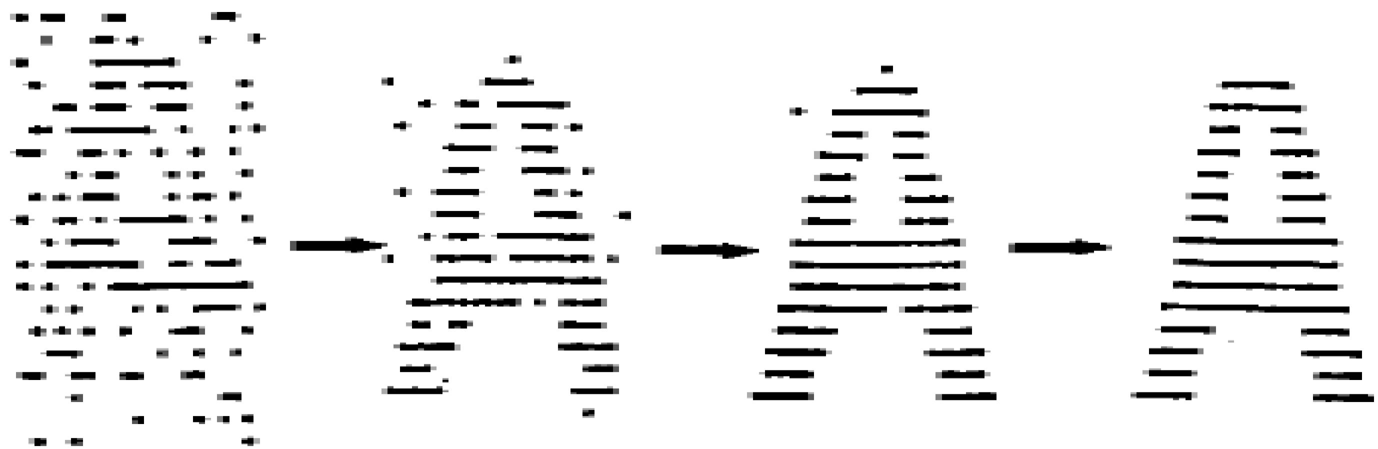
- For each neuron *i*:

  1) compute activation $a_i = \sum_j w_{ij} x_j$

  2) update state of neuron as $x_i = \begin{cases} +1 & a \geq 0 \\ -1 & a < 0 \end{cases}$

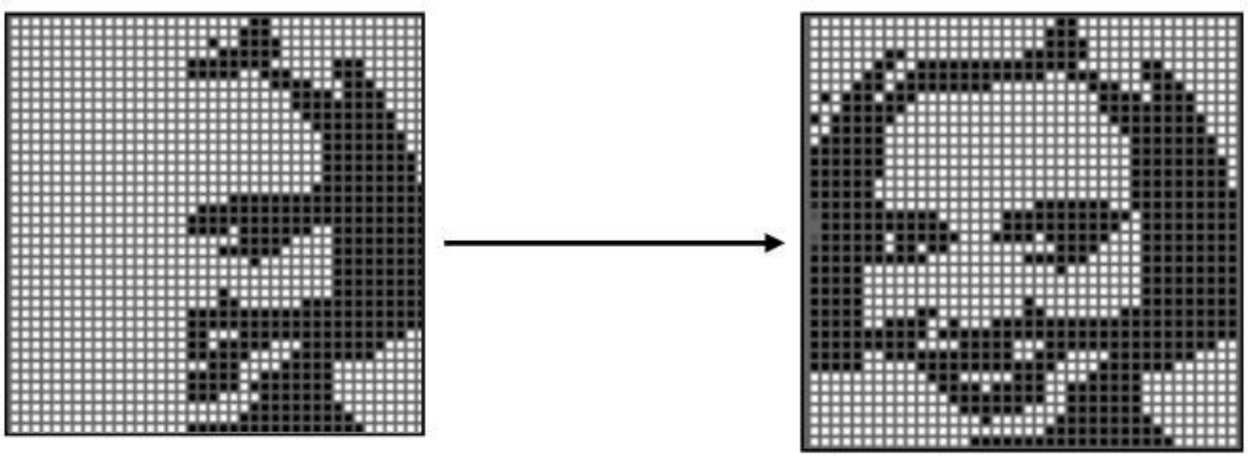
- Updates can be synchronous or asynchronous

Applications: denoising

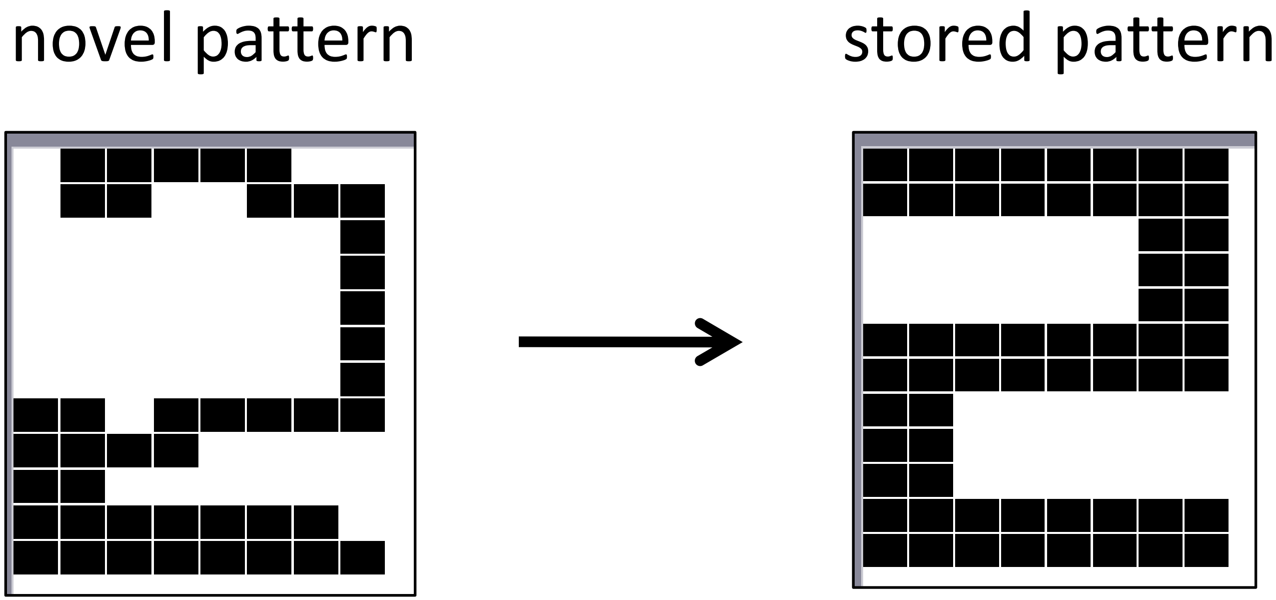


Applications: pattern completion

incomplete pattern complete pattern



Applications: pattern recognition



Issues

- Capacity of the network: how many memories can be stored?
  Not many: $\alpha = M/N = 0.144$
  There is a whole literature about memory capacity and information storage in the hippocampus
- Robustness: how much can we modify a stored pattern such that we recover is perfectly/with a small error?

Hands-on part

- Download exercises from

  http://people.brandeis.edu/~berkes/data/cognitive_models/exercises2