Topics in Combinatorics MATH150B:
The Symmetric Group and Schur Processes

Instructor: Olivier Bernardi

Course description: This class will explore the combinatorics of the symmetric group, and two closely related topics: symmetric functions and probability distributions on integer partitions. The symmetric groups (the groups of permutations) are of great importance because they contain every finite group, and also because of their relations with groups of matrices. The representation theory of the symmetric group is a very rich subject. This theory can be approached using bijections and algorithms on permutations such as the Robinson-Schensted-Knuth correspondence. The symmetric functions also appear naturally in this theory, and again many interesting aspects of symmetric functions can be described in terms of bijections and algorithms. One of the bases of the vector space of symmetric functions in $n$ variables are the Schur functions. The Shur functions are indexed by integer partitions and can be used to define interesting probability measures, called Schur processes, on integer partitions and sequences of integer partitions. Schur processes have nice properties and have interpretations in terms of statistical mechanics model.

In this class we will (tentatively) cover:
- The different bases of symmetric functions.
- Some bits of the general theory of representations of finite groups.
- The irreducible representations of the symmetric group.
- The Robinson-Schensted-Knuth correspondence, and its generalizations.
- The combinatorics of Young tableaux and the connection coefficients in the symmetric group
- The Schur processes and their connections with physical models like the TASEP (totally asymmetric exclusion model).

Requirement: The class is open to highly motivated undergraduate students having already completed classes in linear algebra (MATH15 or MATH22), probability (MATH36A), and algebra (MATH30A or MATH28A+28B). Graduate students in physics or computer science are welcome!