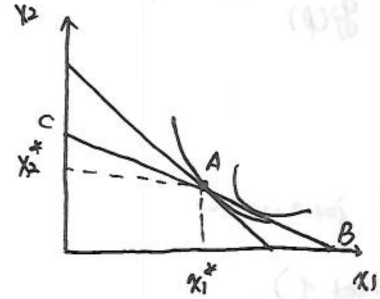
Econ301. Problem Set 5.

1. (a). Under consumption tax on good 1:

$(p_1+t,\ p_2,\ y)$

choice is (x_1^*, x_2^*), such that

$(p_1+t)x_1^* + p_2 \cdot x_2^* = y$



Under income tax $T = t\,x_1^*$

$(p_1,\ p_2,\ y-T) = (p_1,\ p_2,\ y - t x_1^*)$

(x_1^*, x_2^*) is still affordable, because

$p_1 x_1^* + p_2 \cdot x_2^* = y - t x_1^*$

The budget line $p_1 \cdot x_1 + p_2 \cdot x_2 = y - t \cdot x_1^*$ rotates inwards around (x_1^*, x_2^*).

obviously, the consumer will be better off, choosing a point on segment AB, not AC.

(b). $T = y - e[p_1, p_2, v(p_1+t, p_2, y)]$

or $T = e[p_1+t, p_2, u^0] - e[p_1, p_2, u^0]$

$\approx \dfrac{\partial e(p_1, p_2, u^0)}{\partial p_1} \cdot t$ where $u^0 = v(p_1+t, p_2, y)$

2. (a). $e(p,u) = k(u) \cdot g(p)$.

By Shepherd's lemma,

$x_i^h(p,u) = \dfrac{\partial e(p,u)}{\partial p_i} = k(u) \cdot g_i(p)$

$\because x_i(p,y) = x_i^h(p,u)$, where $u = v(p,y)$

$\therefore x_i(p,y) = x_i^h(p,u) = k(u) \cdot g_i(p)$

$\therefore \dfrac{x_i(p,y)}{x_j(p,y)} = \dfrac{k(u) \cdot g_i(p)}{k(u) \cdot g_j(p)} = \dfrac{g_i(p)}{g_j(p)}$ ①

On the other hand,
By Roy's identity,

$x_i(p,y) = -\dfrac{\partial v(p,y)/\partial p_i}{\partial v(p,y)/\partial y}$

$\therefore \dfrac{x_i(p,y)}{x_j(p,y)} = -\dfrac{\partial v(p,y)/\partial p_i}{\partial v(p,y)/\partial y} \cdot (-1) \cdot \dfrac{\partial v(p,y)/\partial y}{\partial v(p,y)/\partial p_j}$

$= -\dfrac{\partial v(p,y)/\partial p_i}{\partial v(p,y)/\partial p_j}$ ②

Combine ①+②, it must be:

$v(p,y) = h(y) \cdot g(p)$

(b). From (a):

$$\frac{x_i(p,y)}{x_j(p,y)} = \frac{g_i(p)}{g_j(p)} \text{ is independent of } y,$$

$\therefore\ y_i = 1,\quad i=1,2\ldots n.$

(Question 3, Pset 1).

3. Let p = price of x_1; y = income

(a).
$$\max_{x_1,x_2} U(x_1,x_2) = u(x_1)+x_2$$
$$\text{s.t.:}\quad p\cdot x_1 + x_2 = y$$

$$\rightarrow L(x_1,x_2,\lambda) = u(x_1)+x_2+\lambda(y-p_1x_1-x_2)$$

FOCs:
$$\frac{\partial L}{\partial x_1} = u'(x_1)-\lambda p = 0$$
$$\frac{\partial L}{\partial x_2} = 1-\lambda = 0$$
$$\frac{\partial L}{\partial \lambda} = y-px_1-x_2 = 0$$

$\Rightarrow\ u'(x_1) = p$

$\Rightarrow\ x_1 = h(p);$

$\Rightarrow\ x_2 = y-px_1 = y-ph(p);$

$$\begin{aligned}\therefore v(p,y) &= u(x_1)+x_2\\ &= u[h(p)]+y-ph(p)\\ &= y+u[h(p)]-ph(p)\\ &= y+F(p)\end{aligned}$$

where $F(p) = u[h(p)]-ph(p)$

(b). $x_1^h(p,u) \equiv x_1[p, e(p,u)]$

In this case,

$x_1[p, e(p,u)] = h(p)$

$\therefore\ x_1^h(p,u) = h(p),$

it means x_1^h does not depend on u.

(c). By Kuhn-Tucker condition:

$$\frac{\partial L}{\partial x_1} = u'(x_1)-\lambda p \le 0,\quad x_1 = 0$$

$\therefore$ when $p \ge u'(0)$, $x_1^* = 0$.

4. Suppose:

Under (p_1^0, t^0, u^0), optimal choice is (x_1^0, x_2^0, x_3^0)

Under (p_1', t', u'), - - - - (x_1', x_2', x_3')

Under (p_1'', t'', u''), - - - - (x_1'', x_2'', x_3'')

↳ where $p_1'' = \alpha p_1^0+(1-\alpha)p_1'$

$t'' = \alpha t^0+(1-\alpha)t'$

$\rightarrow t''p_2 = \alpha t^0p_2+(1-\alpha)t'p_2$

$t''p_3 = \alpha t^0p_3+(1-\alpha)t'p_3$

$u'' = \alpha u^0+(1-\alpha)u'$

$$\begin{aligned}\therefore e^*(p_1'', t'', u'') &= p_1''\cdot x_1''+t''p_2x_2''+t''\cdot p_3\cdot x_3''\\ &= [\alpha p_1^0+(1-\alpha)p_1']\,x_1''\\ &\quad+[\alpha t^0p_2+(1-\alpha)t'p_2]\,x_2''\\ &\quad+[\alpha t^0p_3+(1-\alpha)t'p_3]\,x_3''\\ &= \alpha[p_1^0x_1''+t^0p_2x_2''+t^0p_3x_3'']\\ &\quad+(1-\alpha)[p_1'x_1''+t'p_2x_2''+t'p_3x_3'']\\ &\ge \alpha[p_1^0x_1^0+t^0p_2x_2^0+t^0p_3x_3^0]\\ &\quad+(1-\alpha)[p_1'\cdot x_1'+t'p_2x_2'+t'p_3x_3']\\ &= \alpha\cdot e(p_1^0,t^0,u^0)+(1-\alpha)\cdot e(p_1',t',u')\end{aligned}$$

Therefore, $e(p,t,u)$ is concave in (p,t,u).