Econ301a: Advanced Microeconomics
Fall 2010
B. LeBaron

Final Exam: Solutions

Directions: Do all problems. Put all answers in the blue book. 100 points total. Stay calm and good luck.

I. True, False, Uncertain (Explain) (8 points each)

1. Elasticity of output with respect to a factor is given by

$$\frac{\partial f(x)}{\partial x_i}\frac{x_i}{f(x)}.$$

This equals one when the marginal and average products for factor i are equal.

$$\frac{\partial f(x)}{\partial x_i}\frac{x_i}{f(x)} = MP_i(1/AP_i)$$

TRUE: Obviously, this is one when they are equal.

2. Assume that the expenditure functions for two agents (A and B) are given by

$$e^A(p,u) = k(u)g(p)$$

$$e^B(p,u) = 2e^A(p,u).$$

where $k'(u) > 0$, then the observable market behaviors of A and B are the same.

TRUE

The indirect utility functions for these consumers are,

$$u_A = k^{-1}(\frac{g(p)}{y})$$

$$u_B = k^{-1}(\frac{2g(p)}{y})$$

Since $k'(u) > 0$, $k^{-1}(x)$ is also monotonic, and the two functions would assign the same ranking for any p and y. u_A is some monotonic transform of u_B.

3. A consumer reveals demands for goods 1 and 2 at two different price vectors. For a price vector $(p_1, p_2) = (2, 4)$ the demand is $(q_1, q_2) = (1, 2)$. For a price vector of $(p_1, p_2) = (6, 3)$ the demand is $(q_1, q_2) = (2, 1)$. These choices are consistent with utility maximization.

FALSE

$$y_1 = p_1 1 + p_2 2 = 2 * 1 + 4 * 2 = 10$$

The cost of the other bundle is

$$y = p_1 2 + p_2 1 = 2 * 2 + 4 * 1 = 8$$

Bundle 1 is RP to bundle 2 at the first price vector.

$$y_2 = p_1 1 + p_2 2 = 6 * 2 + 3 * 1 = 15$$

Cost of the first bundle at these prices is,

$$p_1 1 + p_2 2 = 6 * 1 + 3 * 2 = 12$$

So it is in the budget set, and the consumer has shown that bundle 2 is RP to bundle 1. This is violates the weak axiom of RP, and these choices are inconsistent with utility maximization.

II. Short questions: (8 points each)

1. A production technology uses a Leontief format where (K, L) are used in fixed and equal proportions. If $L = 5$, graph the MPK from $K = 0$ to $K = 10$.

$$MPK(K) = 1 \qquad 0 < K < 5$$

$$MPK(K) = 0 \qquad K > 5$$

2. Show that with a constant returns to scale production function, $f(K, L)$, diminishing marginal products for each factor directly implies that that cross partials $f_{K,L} = \frac{\partial f}{\partial K \partial L}$ are positive. (IE. more of L will increase the marginal product of L.)

   From Euler for homogeneous of degree one functions,

$$f(K, L) = K f_K(K, L) + L f_L(K, L)$$

$$f_K = f_K + K f_{K,K} + L f_{K,L}$$

   since

$$f_{K,K} < 0$$

$$f_{K,L} > 0$$

3. A perfectly competitive firm uses energy as one of its factors of production. There are two states of the world which occur with probability (1/2) . In one there is a high energy price $w_{E,H}$, and the other has a low energy price, $w_{E,L}$. The government is considering a policy of smoothing prices (don't ask how), so that firms will pay the expected energy price, $E(w)$ each period. If the firm is risk neutral, and only seeks to maximize its expected profits, $E\pi(p, w)$, each period, will it lobby for or against this policy, or do you not have enough information to tell?

   Profit functions are convex in (p, w), so

$$(1/2)\pi(p, w_{E,H}) + (1/2)\pi(p, w_{E,L}) \geq \pi(p, (1/2)w_{E,H} + (1/2)w_{E,L})$$

   It prefers the fluctuating price.

II. Longer questions

1. (15 points) It is the year, 2020, and Southwest is now the only airline left in the planet's global airline market. It still has fun songs, but its pricing follows that for a regular monopolist. Its demand from business travelers (market 1) is given by $p = 15 - q$, and students (market 2) given by $p = 11 - (1/2)q$. Total costs are given by $C(q) = q$.

   (a) Assume that the firm can perfectly differentiate between the two markets. What price will it charge in each market, and what quantity will it sell in each? What are the profits in each market?

$$MC = 1$$
$$TR_1 = (15 - q_1)q_1$$
$$MR_1 = 15 - 2q_1$$
$$TR_2 = (11 - (1/2)q_2)q_2$$
$$MR_2 = 11 - q_2$$

$$q_1 = 7, \qquad q_2 = 10$$

$$\pi_1 = 8 * 7 - 7 = 49$$
$$\pi_2 = 6 * 10 - 10 = 50$$

   (b) The government would like to tax air travel to pay for increased security. It can only tax one of the two markets with a tax which takes the consumer's price in relation to the producer's to $p_c = p_s + \tau$, with $\tau = 2$. (Hint: Think of the tax as a simple increase in cost from the firm's perspective.) Southwest would readjust output according to this new price. In terms of total revenue which market would it be better for the government to tax? Which market would Southwest prefer to have a tax on?

   Market 1:
   $MR = 3$, $q_1 = 6$, $\pi_1 = 9 * 6 - 3 * 6 = 36$
   Market 2:
   $MR = 3$, $q_2 = 8$, $\pi_2 = 7 * 8 - 3 * 8 = 32$

   Losses will be smaller if the tax is placed on market 1.

2. (8 points) Preferences for two goods are given by quasi-linear utility,

$$u(x_1, x_2) = \log(x_1) + x_2$$

   (a) Find the expenditure function. Assume that good 2 is numeraire with $p_2 = 1$.
   Find Hicksian demands and then expenditure.

$$\min p_1 x_1 + x_2 + \lambda(\log(x_1) + x_2 - u)$$
$$p_1 + \lambda/x_1 = 0$$
$$1 + \lambda = 0$$
$$x_1 = 1/p_1$$
$$x_2 = u - \log(x_1) = u + \log(p_1)$$
$$e(p_1, 1, u) = 1 + u + \log(p_1)$$

(b) *What is the compensating variation of a p_1 increase from 2 to 3? (You can leave stuff like logs in your expression if you have to.)*

$$CV = e(2, 1, u) - e(3, 1, u)$$

$$CV = \log(2) - \log(3) = \log(2/3) < 0$$

(c) *What is the equivalent variation of the same price change?*

$$EV = e(2, 1, u') - e(3, 1, u') = \log(2/3) < 0$$

The sign follows the convention that the price change is a loss to the consumer. Note that the EV is measured at a different utility level, u'. This is the utility level at the income level, y, and the new prices. Because of the quasi-linear structure in the utility function the EV and CV are exactly the same. One way to think about this result is to remember that the CV and EV are integrals under the Hicksian demands, but measured at different benchmark utility levels. In this special form of utility the Hicksian demands for good 1 do not depend on the utility, so EV and CV are the same. The property leads to a lot of uses for this form a utility because of its convenience, but be aware of its very special restrictions that it places on behavior.

3. (7 points) In the standard general equilibrium framework we assumed strong monotonicity for preferences. For this problem weaken this, and allow consumers to be in the interior of the budget sets,

$$p \cdot x^i \leq p \cdot w^i.$$

In this case can you develop a weaker version of Walras' Law? Would it allow for a situation where all prices are strictly positive, and one market has an excess demand?

$$p \cdot (x^i - w^i) \leq 0$$

Summed over everyone gives,

$$p \cdot z \leq 0$$

where z is the excess demand. This is a weaker version of Walras' law which states this with an equality. That situation would require at least one market to have an excess supply. Nothing in the assumptions rules this out, so that situation is possible. (Note: strong monotonicity in preferences would rule out the excess supply possibility.)

4. (15 points) Assume people have CARA preferences

$$u(w) = \frac{-1}{\gamma} e^{-\gamma w}$$

For normal random variables $E(e^{\tilde{x}}) = e^{\mu + (1/2)\sigma^2}$ where $x \sim N(\mu, \sigma^2)$. You don't need to derive this, but the above preferences are then equivalent to

$$E(u(\tilde{w})) = E(\tilde{w}) - \gamma \sigma_w^2 \qquad \sigma_w^2 = var(\tilde{w})$$

Use these preferences to determine whether they should take certain gambles which add to their wealth as follows,

$$\tilde{w} = w + \tilde{z}$$

$$E(z) = 0, E(z^2) = \sigma^2$$

Agents will compare receiving wealth plus the gamble, $\tilde{w}$, versus wealth minus a premium paid to avoid the gamble, $w - \pi$.

(a) If you observe a consumer turning down a gamble z, and paying a premium, π_1, what can you infer about the magnitude of γ?

$$w - \pi_1 > E(\tilde{w}) - \gamma \sigma_w^2$$

$$w - \pi_1 > w - \gamma \sigma_w^2$$

$$w - \pi_1 > w - \gamma \sigma_w^2$$

$$\gamma > \frac{\pi_1}{\sigma_w^2}$$

(b) This person is now faced with a larger gamble with

$$E(z) = 0, E(z^2) = 5\sigma^2.$$

If π_2 is the premium for avoiding this gamble what is the largest value of π_2 that would cause the least risk averse person to chose to avoid the gamble?

$$w - \pi_2 > E(\tilde{w}) - \gamma 5 \sigma_w^2$$

$$\pi_2 < \gamma 5 \sigma_w^2$$

$$\pi_2 < \frac{\pi_1}{\sigma_w^2} 5 \sigma_w^2$$

$$\pi_2 < \pi_1 5$$

(c) Could there be values for w and π_1 for which the consumer would always pay to avoid the larger gamble? (Assume you can't pay more than w.)

$$E(\tilde{w}) - \gamma 5 \sigma_w^2 < 0$$

This means that the consumer would always prefer to pay down all wealth to zero, and would pay anything to avoid the gamble.

$$w - 5\pi_1 < 0$$

This is the condition for the least risk averse person according to gamble 1.

5. (7 points) What is framing? How does it impact people's decisions under uncertainty? Why might it be something that is easy to address through policy changes?

   Framing tells us that individuals may be impacted by the way information is presented. This may be especially true when uncertainty is involved. In many experiments, people have been presented the same information, but in different ways, and this has lead to different decisions. Framing can be very important for policy makers since changing the wording, or presentation of information, is a very cheap adjustment, which could have very big impacts. For many examples see the book Nudge.