Econ301a: Advanced Microeconomics
Fall 2010
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Midterm Exam: Solutions

Directions: Do all problems. 100 points total. Stay calm and good luck.

I. True, False, Uncertain (Explain) (5 points each)

1. Suppose a consumer's uncompensated own price elasticity for good X is less than 1 in absolute value, and you know the good is not a Giffen good. The consumer buys many goods. A rise in the price of X (only X) must reduce his demand for some other good. The basic idea is this problem is to figure

   out what happens to the total amount spend on good X, $p_X X$. Since this is not a Giffen good the quantity X falls. However, the value spent on X goes to

   $$d(P_X X)/dP_X = (X + P_X dX/dP_X) = (1/X)(1 + \epsilon_{XX})$$

   Since elasticity is less than 1 in absolute value, the above derivative is positive. So the total amount on X rises, and therefore the amount spent on some other good must fall.

2. A worker is a net supplier of hours to the labor market. A given worker is at an interior solution (works somewhere between 0 and 24 hours per day). An increase in the wage will always cause the worker to increase hours worked.

   This is a simple income/substitution question. Since these go in opposite directions, you don't now the final sign as to whether work more or less.

3. The same worker is now offered an overtime wage. This gives the worker the current wage for the hours the worker is currently working, and a higher wage for any additional hours the worker might chose. If a worker is offered an overtime wage above the current wage he or she will always work more hours.

   This is actually true. This is an interesting modification of the last question. There is actually no income effect, since the consumer is not wealthier at the current hours worked. However, it tilts the relative value of work and leisure for all hours worked beyond those currently worked. If the worker was at an interior FOC, then this will cause the worker to work more. (A drawing of this would include a kinked budget set which changes slope at the current choice of hours.)

4. Consumers will always be at least as well off with Slutsky compensation as with Hicksian compensation.

   True: Hicksian compensation puts a consumer back on the original indifference curve, but Slutsky just makes sure that the original bundle is in the new budget. So the Hicks compensated consumer is always at least as well off as before, and possibly better off.

5. If I know a consumer's *money metric utility function*, and current consumption bundle, then I would be able to calculate how much to compensate the consumer for a price change p^0 to p^1 leaving the consumer at the same utility as before the price change.

   Since the money metric utility function tells you the cost of getting the utility of a given bundle, at a given set of prices, it can tell you how much it will cost to keep a consumer at the utility level they are currently at for a new set of prices.

   $$\mu(p^1, q^0)$$

   would tell you how much it costs to be as well off as bundle q_0 at prices p^1.

II. Short questions (8 points each)

1. Consider a world where a consumer can receive x_H in a good state of the world with probability p, and x_L in the bad state of the world with probability $(1-p)$. Define the certainty equivalent consumption x_p as that level of consumption which leaves the consumer indifferent between the gamble and and x_p with certainty. Compare this with a second gamble over x_H and x_L with probability q of the H state, and $1-q$ for the L state. For this gamble there is a certainty equivalent x_q. The consumer has well a well defined expected utility function over the gambles. Show that

$$\frac{u(x_p)-u(x_q)}{u(x_H)-u(x_L)}$$

depends only on the difference between the probabilities p and q.

$$u(x_p) = pu(x_H) + (1-p)u(x_L)$$

$$u(x_p) = qu(x_H) + (1-q)u(x_L)$$

$$u(x_p) - u(x_q) = u(x_H)(p-q) - u(x_L)(p-q)$$

2. A consumer consumes only two goods, 1 and 2. Assume that $p_1 = p_2$, and $x_1 = x_2$. Let $\epsilon_{i,j}$ be the uncompensated elasticity of demand for good i, with respect to price j. Also, let η_j represent the income elasticity for good j. Show that $\epsilon_{1,2} - \epsilon_{2,1} = 0.5(\eta_2 - \eta_1)$.

$$\epsilon_{1,2} = \epsilon_{1,2}^c - s_2\eta_1$$

Since $\partial x_1/\partial p_2$ equals $\partial x_2/\partial p_2$ and the quantities and prices are equal, $\epsilon_{1,2}^c = \epsilon_{2,1}^c$, and therefore,

$$\epsilon_{1,2} - \epsilon_{2,1} = 0.5(\eta_1 - \eta_2)$$

Also, since $s_1 = s_2 = 0.5$.

3. Assume that preferences over n goods are additive and given by

$$U = \sum_{i=1}^{n} u_i(x_i).$$

You know that $u_i'(x_i) > 0$, and $u_i''(x_i) < 0$. Show that these preferences imply that there are no inferior goods, $\partial x_i(p, y)/\partial y \geq 0$.

From consumer optimization we have,

$$u'(x_i)/u'(x_j) = p_j/p_i$$

Assume that good i is inferior. An increase in income (no price changes) means that the consumption of good i will fall. However, this above relationship shows that the consumption of good j must also fall. One can then continue this logic for goods j and k. This would imply that consumption of k must now fall. Continue on, and you get that consumption of all goods must fall, which is clearly impossible since they all can't be inferior.

4. Consumers are faced with a new gasoline tax which is charged per gallon of gas, but the government refunds all the tax expenditures to each consumer as in,

$$(p+t)x_g + x = y + tx_g.$$

x_g is the amount of gasoline consumed, and x represents all other goods with price equal 1. What is the impact on the demand for gas $x_g(p+t, 1, y)$ of a small increase in t starting from $t = 0$?

$$dx(p+t, y+tx_g)/dt = \partial x(p,y)/\partial p + \partial x(p,y)/\partial y x_g$$

Now, remembering the Slutsky equation it is easy to see that the left side of this is equal to

$$\partial x^h(p,y)/\partial p$$

So the impact would be just the same as the Hick's compensated change in demand for small t near $t = 0$.

5. Let $x_i(p, y)$ be the standard Marshallian demand for x. Define the indirect demand implicitly as $x = (g(x), 1)$. $g(x)$ gives the price vector that sets demand to x for $y = 1$. Show that

$$g_i(x) = \frac{1}{\sum_{i=1}^{n} x_i u_i(x)} u_i(x)$$

where $u_i(x) = \partial u(x)/\partial x_i$ and $g_i(x)$ corresponds to p_i

The FOC for each good gives,

$$u_i(x) = \lambda p_i$$

Multiply by x_i.

$$u_i(x)x_i = \lambda p_i x_i$$

$$\sum_{i=1}^{n} u_i(x)x_i = \lambda \sum_{i=1}^{n} p_i x_i$$

Since $y = 1$ we get,

$$\sum_{i=1}^{n} u_i(x)x_i = \lambda$$

Plug this into the first order condition for i, solved for p_i, to get the $g_i(x)$ result.

6. Preferences for two goods are given by quasi-linear utility,

$$u(x_1, x_2) = \log(x_1) + x_2$$

Graph the Marshallian demand for good 2 as a function of p_2 for a given level of income y.

II. Longer problems:

1. (9 points) A worker can receive two possible wage levels next year w_L, a low wage, with probability p, and w_H, a high wage, with probability $1-p$. The worker may purchase insurance over these states of the world by paying a premium for a payment that will occur in the bad state of the world. Specifically, the consumer can purchase a policy that will pay q in the bad state by paying πq in the good state of the world. π is the premium which is essentially the price of insurance. Now further assume that the utility function for the consumer is given by,

$$U = p\log(w_L + q) + (1-p)\log(w_H - \pi q)$$

For which prices, π, would the consumer decide to "self-insure"? That is set $q = 0$.

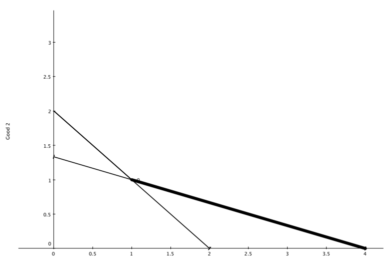
$$dU/dq = \frac{p}{w_L + q} - \frac{\pi(1-p)}{w_H - \pi q} = 0$$

Now set $q = 0$, and solve for π.

$$p(w_H) = \pi(1-p)(w_L)$$

$$p(w_H)/((1-p)(w_L)) = \pi$$

2. (9 points) A consumer currently is choosing consumption bundles from two goods. Currently the price of the two goods is $p_1 = p_2 = 1$, and the income of the consumer, $y = 2$. The consumer purchases a (x_1, x_2) bundle of $(1, 1)$. Now the prices change to $(p_1, p_2) = (0.5, 1.5)$. Draw this situation. If we assume that the consumer will stay on the budget line, and the weak axiom of revealed preference holds, then show which portion of the budget line the consumer will be on. (This could be the entire line.) Does this picture tell you anything about Slutsky compensated demand functions?



The darker portion of the line is the area where the consumer can go.

This figure can also be viewed as a compensated demand experiment for an increase in the price of good 2 with an income compensation to make the original bundle affordable. This clearly shows that in that situation the demand for good 2 would fall. Compensated demand slopes down.

3. (9 points) In the paper "Giffen behavior and subsistence consumption" the authors used a special mechanism to simulate a price decrease for consumers. What was this mechanism? Why didn't they just observe the response of consumers' demands to price changes in the market over time?

The paper used a voucher system to simulate an exogenous price change. The need is to expose some subset of consumer to a change in price which is independent of all other economic events. Regressing demand on changes in the market price is never a good idea in economics since this will involve changes in demand, and supply, and probably many other things. Also, in this case the price (on the right hand side of the regression) is endogenous to the system. The vouchers create an artificially exogenous variable.