Econ301a: Advanced Microeconomics
Fall 2008
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Midterm Exam

Directions: Do all problems. 100 points total. Stay calm and good luck.

I. True, False, Uncertain (Explain) (8 points each)

1. If all income elasticities are constant and equal, then they must be equal to 1.

   TRUE:

$$\sum p_i x_i = y$$
$$\sum p_i \frac{\partial x_i}{\partial y} = 1$$
$$\sum p_i (x_i/y) \frac{y \partial x_i}{x_i \partial y} = 1$$
$$\sum s_i \eta_i = 1$$
$$\sum s_i \eta = 1, \eta_i = 1$$

2. Assume a world with only lotteries over two outcomes, a_1 and a_2. In this world an expected utility function is given by

$$U = pu(a_1) + (1-p)u(a_2).$$

   In this world, it is true that any monotonic transformation of this utility function will give the same ranking on all gambles. You can assume that preferences follow all axioms for choice under uncertainty.

   TRUE: This is (of course) not true in general, but for the special case of two gambles it is true.

   Assume that $u(a_1) > u(a_2)$ since one must be greater.

   Define the gamble, g_p as

$$(p \circ a_1, (1-p) \circ a_2)$$

   Axiom G4 give,

$$g_x \succeq g_y \qquad iff \qquad x \geq y$$

   Consider a new utility ranking given by $V(u(a_j))$ where $V()$ monotonic.

$$V(u(a_1)) > V(u(a_2))$$

   Now expected utilities

$$E(u) = pu(a_1) + (1-p)u(a_2)$$
$$E(V) = pV(u(a_1)) + (1-p)V(u(a_2))$$

   All we need is for

$$\frac{dV(x)}{dx} > 0$$

   to maintain the ranking in p from the original preferences. This is clearly true from the above inequality on V.

3. The expenditure function for consumer A is given by $e^A(p,u) = k(u)g(p)$ and for consumer B by $e^B(p,u) = 2e^A(p,u)$. A and B are indistinguishable using observable market data.

   TRUE

   First, $k()$ must be a monotonic function. Therefore so is, $k^{-1}()$. Apply the following transformation to person A's utility,

   $$u_2 = k^{-1}(2k(u))$$

   This is monotonic and would represent the same observables as for u. Plugging this into A's expenditure function would give B's.

   Think about transforming the utility of person A by $k^-1(u)$. This would give a consumer with expenditure $ug(p)$

4. When the ratio of all goods consumed x_i/x_j is independent of the level of income for all i and j, then all income elasticities must be equal.

   TRUE

   $$\sum p_i x_i = y$$

   Pull out good 1 as a kind of representative good.

   $$\frac{\sum p_i x_i}{p_1 x_1} = \frac{y}{p_1 x_1}$$

   Holding prices fixed, the left side is constant, and therefore,

   $$x_i = a_i y$$

   for any rep good i.

   The income elasticity for good i is then,

   $$\frac{y}{x_i} dx_i dy = a_i(y/x_i) = 1$$

   So they are not only all equal, they are one.

5. If a consumer has additively separable preferences,

   $$U = \sum_{i=1}^{n} u_i(x_i)$$

   then the marginal rate of substitution between goods i and j depends only on x_i and x_j. Therefore, the Marshallian demand for x_i does not depend on p_k, the price of a third good, different from i and j.

   FALSE

   Demands for good i is also affected by price of good k through the income effect. Consumers would clearly consume i and j in the same proportion, but income available would change as p_k changes, and the demand for the two goods i and j would change.

6. If there are three goods and x_1 is a substitute for x_2 in the Hicksian demand sense, and x_2 is a complement for x_3, then x_3 must be a substitute for x_1.

   TRUE

   Remember first that all own effects are negative. These are all Hicksian demands.

$$\partial x_1/\partial p_1 < 0 \qquad \partial x_2 \partial p_2 < 0 \qquad \partial x_3/\partial p_3 < 0$$

Symmetry and the info in the problem gives

$$\partial x_1/\partial p_2 > 0 \qquad \partial x_2/\partial p_1 > 0 \qquad \partial x_3/\partial p_2 < 0$$

Hicks third law

$$\sum_{j=1}^{n} \frac{\partial x_i(p,u)}{\partial p_j} p_j = 0$$

Shows that at least one partial must be positive.

Doing this for $\partial x_3/\partial p_j$ gives that

$$\partial x_3/\partial p_1 > 0$$

II. Longer problems

1. (12 points) Brandeis is worried about parking violations. The parking office is considering increasing the number of people checking stickers. A parking ticket currently costs an amount, F. The probability p represents the chance of getting a ticket. G is the gain from parking illegally (which you get regardless of whether you get caught). Brandeis' policy will increase p, but they will change F, so that $pF = k$ stays constant. Assume that parkers maximize expected utility, and are risk averse, $u''(x) < 0$, with a wealth level, w. Also, assume there is some amount of illegal parking taking place at the current level p. Can you tell what the impact will be from an increase in p on illegal parking at Brandeis?

   This problem has two pretty good solution methods. The first is graphical

   The second views the expected utility in a Taylor series approximation.

$$w_g = w + G$$

$$w_b = w + G - F$$

Gamble

$$(w + G) + \tilde{z}$$

Good (1-p)

$$\tilde{z} = 0$$

Bad (p)

$$\tilde{z} = -F$$

$$E(\tilde{z}) = (1-p)0 - pF$$

No change in expected value as the policy changes since pF is fixed.

Expected utility can be estimated as a Taylor series as in

$$E(u(w + G + \tilde{z})) = u'(w + G)E(\tilde{z}) + (1/2)u''(w + G)E(\tilde{z}^2)$$

Since the $E(\tilde{z})$ doesn't change we need to know what happens to $E(\tilde{z}^2)$ as p is increased.

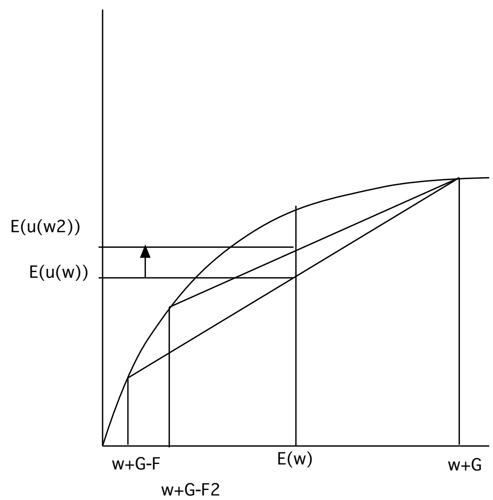
$$E(\tilde{z}^2) = (1-p)0 + p(F)^2$$

$$E(\tilde{z}^2) = p(k/p)^2 = k/p$$

The latter must fall as p increases, so the second moment of $\tilde{z}$ will fall. This raises the expected utility of parking illegally ($u''(w) < 0$), and will have the effect of causing more illegal parking.

This problem can also be done graphically. See JR page 106 for a good example.

Basically, the usual plot applies. The middle point represents the expected wealth level, and it doesn't change. The intersection of this with the lines connecting the endpoints on the utility plots shows the expected utility. The only change is that the left wealth level shifts to the right as the penalty falls to balance the rise in p. For any risk averse (concave) preferences, expected utility will rise, and there will be more illegal parking.



2. (20 points) Recently social security recipients in the US have received a cost of living adjustment, increasing their benefits to compensate them for price increases. Many of these have been energy related. Assume that consumers consume two goods, 1 and 2.

   (a) If consumers are compensated by boosting their income in proportion to an increase in their current "cost of living",

   $$y' = y\left(\frac{p_1' x_1 + p_2' x_2}{p_1 x_1 + p_2 x_2}\right)$$

   where (x_1, x_2) is their current bundle, can you tell if they will be worse off after this change?

   If they are compensated in this fashion, then they can always consume their previous bundle, so they can't be any worse off.

   (b) In the following parts, consider 3 different price changes

   $$\begin{aligned} p &= (p_1, p_2) \\ p^a &= (p_1 + 2\Delta p, p_2) \\ p^b &= (p_1, p_2 + 2\Delta p) \\ p^c &= (p_1 + \Delta p, p_2 + \Delta p) \end{aligned}$$

   Assume that Δp is small (you can use Taylor series approximations). If you know that $x_1 > x_2$, can you show locally, whether consumers would prefer p^a or p^b (with no compensation)?

   Use the indirect utility function,

   $$v(p_1, p_2, y) = \max u(x_1, x_2) + \lambda(y - p_1 x_1 - p_2 x_2)$$

   Envelope thm gives

   $$\frac{\partial V}{\partial p_1} = -\lambda x_1$$

   $$\frac{\partial V}{\partial p_2} = -\lambda x_2$$

   The approximate change in utility from a first order Taylor would then be

   $$\Delta V = -\lambda x_i 2\Delta p$$

   This is obviously a larger fall in utility for a change in p_1, so the change in p_2 is preferred.

   (c) Now drop the assumption that Δp is small (no Taylor series approximation). Also, assume the following compensation levels for each price change,

   $$\begin{aligned} y^a &= y + 2\Delta p x_1 \\ y^b &= y + 2\Delta p x_2 \\ y^c &= y + \Delta p(x_1 + x_2). \end{aligned}$$

   Can you show that one of the three price changes + compensation could **not** be strictly preferred to the other two?

   Price change c is a convex combination of the other 2.

   $$(p^c, y^c) = (1/2)(p^a, y^a) + (1/2)(p^b, y^b)$$

   From quasi convexity in the indirect utility we know that

   $$v(p^c, y^c) \leq \max(v(p^a, y^a), v(p^b, y^b))$$

   So price change and income compensation c could not be preferred to both a and b.

3. (20 points) An investor decides on the amount of wealth, x to invest in a risky asset. The return on this asset in the good state of the world is given by r_g, and in the bad state of the world by r_b. The probability of the good state is p, and the bad state is $1-p$. Assume that wealth that is not invested does not change from today to the future. Wealth in the two states is then,

$$w_g = w - x + x(1 + r_g)$$

and

$$w_b = w - x + x(1 + r_b)$$

The investor maximizes expected utility, $Eu(\tilde{w})$, of future wealth. For all parts of the problem, you know that $u'(w) > 0$ for all w.

(a) What are the first and second order conditions for x?

$$w_g = w + xr_g$$

$$w_b = w + xr_b$$

$$E(u) = pu(w + xr_g) + (1 - p)u(w + r_b)$$
$$0 = E(du/dx) = pu'(w + xr_g)r_g + (1 - p)u'(w + xr_b)r_b$$
$$E(d^2u/dx^2) = pu''(w_g)r_g^2 + (1 - p)u''(w_b)r_b^2 < 0$$

(b) If the investor is risk averse, $u''(w) < 0$, then show that for any investment with an expected return, $E(r) < 0$, the investor will set $x = 0$. Can you also show that this same investor will invest more than $x = 0$ if the expected value of the investment is positive?

Eval at $x = 0$

$$dE(u)/dx = pu'(w)r_g + (1 - p)u'(w)r_b$$
$$dE(u)/dx = u'(w)(pr_g + (1 - p)r_b) = u'(w)E(r) < 0$$

So locally increasing x will decrease utility. Also, concavity (the 2nd order) makes sure that $dE(u)/dx$ will only decrease as x increases from zero. The optimal investment will have to be $x = 0$.

(c) What happens to the level of the investment, x, when the return is taxed. The after tax return is now $(1-t)r_g$, and $(1-t)r_b$. Assume you know the optimal level of investment, x^* without the tax. Now find the optimal level of investment with the tax, $\hat{x}$, in terms of x^*.

$$0 = pu'(w + (1 - t)xr_g)(1 - t)r_g + (1 - p)u'(w + (1 - t)xr_b)(1 - t)r_b$$

$$0 = pu'(w + (1 - t)xr_g)r_g + (1 - p)u'(w + (1 - t)xr_b)r_b$$

If x^* solved the previous FOC's. Then $\hat{x}(1-t) = x^*$ will solve this FOC.

$$\hat{x} = 1/(1 - t)x^*$$

(d) In the case of a risk averse investor with $x^* > 0$ can you tell if investment increases or decreases after the tax?

Investment actually increases after the tax. $\hat{x} > x^*$.