

Problem Set 10: Due Wednesday, Dec. 1

1. True, False, Uncertain: Explain
 - (a) A profit maximizing monopoly will always produce an output lower than that which would maximize its sales revenue.
 - (b) If you know a market has a constant elasticity market demand function, then you would be able to tell the percentage price markup over marginal cost without knowing anything else about the firm's production technology.
2. Consider two firms selling passenger jet aircraft. They are located in different countries, and sell into the world market. The two firms are Boeing and Airbus. The world demand for jets is given by

$$p = 20 - q$$

Both firms produce with the same linear total cost function,

$$c_1(q_1) = 8q_1$$

$$c_2(q_2) = 8q_2$$

- (a) First consider the market where there is only one producer. Find the monopoly quantity, price, and profits for the monopolist.
- (b) Now assume both firms are producing. Find the Cournot/Nash equilibrium for these two firms. Remember that $q = q_1 + q_2$, and each firm will maximize its own profits assuming the other one stays with its current output level. Record the amount produced by each firm, the price, and the profits for the two firms.
- (c) How has price and output changed from the monopoly situation? Would these firms prefer to perfectly collude and behave as a monopolist, splitting the monopoly profits?
- (d) Now the government in the country where Boeing resides decides to subsidize the production of jets. They put on a per unit subsidy of 3 which reduces the effective MC to $8 - 3 = 5$. There is no change in the country in which Airbus resides. Find the new Cournot/Nash equilibrium, and report the price, and profits. Has the price fallen or increased?
- (e) Report the profit net of the subsidy amount. Has this increased relative to the case w/o the subsidy? If the government lump sum taxes the subsidy cost from the firm, and cares only about the firms profits, and the well being of the airline industry purchasing these jets, will all 3 of these parties prefer the case with the subsidy.
- (f) Why is this subsidy making this country better off? What aspect of the Cournot assumption of the firm is not right?

3. An interesting measure related to factor demands is the share of marginal cost attributed to factor j . This can be written as,

$$\frac{w_i}{MC} \frac{\partial x_i}{\partial y}$$

- (a) Why does this make sense?
- (b) Show that this is equal to the elasticity of MC with respect to the price of factor i , w_i .
- (c) Use the results from cost functions for homothetic production functions to get a sign for the last elasticity.
- (d) Now use results on the profit function to determine a simple relationship between

$$\frac{p}{x_i} \frac{\partial x_i}{\partial p}$$

and

$$\frac{w_i}{y} \frac{\partial y}{\partial w_i}$$

4. A firm has two plants, one with cost $c_1(y) = (1/2)y^2$, and the other with cost $c_2(y) = y$. What is the cost function for this firm?

The firm will produce each new unit of the good at the lowest MC plant. This means that it will operate plant one $MC = y$ for units 0 to 1, and then plant 2, $MC = 1$ for all units thereafter. Total cost is then $(1/2)y^2$ for $y < 1$, and $1/2 + (y - 1)$ for $y > 1$

5. A firm has a production function given by,

$$f(x_1, x_2) = \min(2x_1 + x_2, x_1 + 2x_2)$$

What is the cost function for this technology?

6. If the wage of labor rises relative to the cost of capital the share of labor costs in total production costs will rise if and only if the elasticity of substitution is less than one.