ECON301a: Advanced Microeconomics
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Problem Set 2: Due Wednesday, Sept 15th

1. In class (and in Miller's notes) we build the substitution matrix as,

$$s_{ij} = \frac{\partial x_i(p,y)}{\partial p_j} + \frac{\partial x_i(p,y)}{\partial y} x_j^0 = \frac{\partial x_i^c(p, p \cdot x^0)}{\partial p_j}$$

$$dx_i^c = s_{ij} dp_j$$

   The total differential would give,

$$dx_i^c = \sum_j s_{ij} dp_j$$

$$s_i = (s_{i1}, s_{i2}, \ldots, s_{iL})$$

   S is made of of the rows s_i. Show that $Sp = 0$, and $p \cdot S = 0$. The first of these is a standard matrix multiply yielding a vector of zeros. This second is the dot product applied from p to S which gives

$$\sum_{i=1}^{L} p_i \frac{\partial x_i^c(p,y)}{\partial p_j}$$

   for all goods j. Show this entire vector is zero for each j.

2. Assume axioms 1 and 2 (completeness and transitivity) hold for the "at least as good" relation, $\succeq$. (Do not assume other axioms.) Define the "strictly preferred", $\succ$, and the indifferent, $\sim$, as in 1.2 and 1.3 from the book. Show that transitivity holds for both these operators. ($x \succ y$ and $y \succ z \rightarrow x \succ z$), ($x \sim y$ and $y \sim z \rightarrow x \sim z$.)

3. Briefly show that axiom 4 is stronger than axiom 4'.

4. J and R, problem 1.9. Strict monotonicity is defined differently in some books. For example, see Varian page 96. Would these preferences satisfy Varian's strong monotonicity axiom? Varian adds to the axiom the fact that if a bundle x has at least as much of all goods as a bundle y, and is not equal to bundle y, then $x \succ y$.

5. Axiom 3 (page 8) seems somewhat nonintuitive in terms of economics, but it turns out to be pretty important for building utility functions. The proof of the existence of a utility function with just axioms 1-3 is very difficult and beyond the scope of the course. However, it is not that hard to show that axiom 3 is *necessary* for the existence of a well behaved utility function. Assume you have a continuous utility function, $u : \Re_+^n \rightarrow \Re$. Show that axiom 3 is necessary for the existence of this function. (Explain what necessity entails, and first where you are going with your proof strategy.) (Hint: You can freely use Thm A1.9 in your proof.)