

**Problem Set 3: Due Wednesday, Sept 22**

1. You will often hear a reference to Lexicographic preferences in economics. In this case a consumer ranks the goods in a bundle in order of importance, and begins checking over each moving down this ranking. The consumer stops when two amounts are not equal, the one with more is the preferred bundle. For example, with two goods, the bundle  $(1, 2)$  is preferred to  $(1, 1)$  since  $2 > 1$ . However,  $(3, 1)$  is preferred to  $(1, 5)$  since  $3 > 1$ . The idea is that you move across the goods until you get an inequality. Show that these preferences violate Axiom 3 (continuity), and also cannot have a continuous utility function which represents them. In two good space, plot an arbitrary bundle  $A$ , and divide the space in to the better, worse, and indifference bundles for these preferences. (This is similar to JR 1.13.)
2. For almost all of our results on consumer optimization we require that the consumer be in the interior of their budget set. This follows from Lagrange's Thm, A2.16. For our necessary conditions to be necessary we can't be on the boundary of what is feasible. This implies that all goods must be consumed in strictly positive quantities. Can you draw valid preferences and prices in  $(x_1, x_2)$  space for which one of the two goods is not consumed? For your picture do the usual first order conditions hold? If not, can you state any inequalities for them? (Eventually, we will move on to the Kuhn-Tucker conditions which will help us in these cases.)
3. Consider the utility function given by,

$$u(x_1, x_2) = x_1^{1/3} x_2^{1/3}$$

- (a) Is this concave or convex? Strictly?
- (b) Apply the monotonic transformation,  $y = x^6$ . Now is the function concave or convex?
- (c) If the function in the first part was quasi-concave, what about the function in part 2?

**Answer True, False, Uncertain (and explain)**

4. In a multi good world, if two goods are consumed in equal quantities,  $x_i = x_j$ , and have equal income elasticities of demand,  $\eta_i = \eta_j$ , then they will exhibit symmetry in the *Marshallian* demands.

$$\frac{\partial x_i(p, y)}{\partial p_j} = \frac{\partial x_j(p, y)}{\partial p_i}$$

5. If a consumer has additively separable preferences,

$$U = \sum_{i=1}^n u_i(x_i)$$

then the marginal rate of substitution between goods  $i$  and  $j$  depends only on  $x_i$  and  $x_j$ . Therefore, the Marshallian demand for  $x_i$  does not depend on  $p_k$ , the price of a third good, different from  $i$  and  $j$ .