

Problem Set 4: Due Friday, Sept 29th

1. True, False, Uncertain : explain

- (a) The price of food rises by 10% and disposable income by 5%. A person initially spending half their income on food would be neither better nor worse off by these changes. (Make sure you give me a picture on this problem.)
- (b) If a consumer's indirect utility function is given by

$$\frac{g(p_1, p_2, \dots, p_n)}{y} + h(p_1, p_2, \dots, p_n)$$

where p_i is the price of good i , and y is income, then the function $g(p_1, p_2, \dots, p_n)$ must be homogeneous of degree one, and the function $h(p_1, p_2, \dots, p_n)$ must be homogeneous of degree zero.

2. Here is a slightly annoying problem, but it isn't all that hard. An often used form for utility is known as the constant elasticity of substitution utility function. It is given by,

$$u(x_1, x_2) = (x_1^\rho + x_2^\rho)^{1/\rho}$$

You know $0 < \rho < 1$.

- (a) I want you to set up the Lagrangian for this problem. Is there a trick that might make your life easier in working out this problem?
- (b) Use this trick (hopefully you got it) to find the Marshallian demands for good 1 as a function of the two prices and income, y .
- (c) Check that this function is homogeneous of degree 0 in (p, y) .
3. In class we talked about the Cobb/Douglas form of utility functions given by,

$$u(x_1, x_2) = x_1^\alpha x_2^{1-\alpha}$$

This would appear to be a kind of restricted form of

$$u(x_1, x_2) = x_1^\gamma x_2^\beta.$$

Show in the world of utility functions that the first restriction doesn't matter, and that there is an α which represents the (γ, β) form. Can you find this α in terms of (γ, β) ? You can assume that $\alpha, \gamma, \beta > 0$.

4. Homothetic preferences play a crucial role in economics. They are defined in the following way. A utility function is homothetic if

$$u(x) = f(g(x))$$

where $f(z)$ is a monotonic transformation, and $g(x)$ is homogeneous of degree 1.

- (a) Analyzing these preferences can you just consider $g(x)$, why or why not?
- (b) Show that the expenditure function is of the form $e(p, u) = e(p)u$.
- (c) Show that the indirect utility function is of the form $v(p, y) = v(p)y$.
- (d) Show that the Marshallian demands must be of the form $x_i(p, y) = x_i(p)y$. (It is clear from this that the "income expansion path" for these goods must be linear in y , and the ratio of any two goods consumed x_i/x_j is independent of income.)
- (e) Show that the income elasticity of demand for any good is constant over different income levels.