Problem Set 6. Econ301. Fall 2010

1. JR 1.65

(a). If $e(p, u^0) \uparrow$,

$e(p', u^0) > e(p^0, u^0)$

then $I(p', p^0, u^0) \equiv \frac{e(p', u^0)}{e(p^0, u^0)} > 1$

(b). By definition,

$$\frac{y'}{y^0} = \frac{e(p', u')}{e(p^0, u^0)}$$

If $\frac{y'}{y^0} > I(p', p^0, u^0)$

then $\frac{e(p', u')}{e(p^0, u^0)} > \frac{e(p', u^0)}{e(p^0, u^0)}$

then $e(p', u') > e(p', u^0)$

$\because$ $e(p, u)$ is increasing in u

$\therefore$ $u' > u^0$

1. JR 1.66

(a). $\max_{x_1, x_2} u(x_1, x_2) = \sqrt{x_1} + x_2$

s.t: $1 \cdot x_1 + 2 \cdot x_2 = 10$

$L(x_1, x_2, \lambda) = \sqrt{x_1} + x_2 + \lambda(10 - x_1 - 2x_2)$

FOCs:
$$\left.\begin{aligned} \frac{\partial L}{\partial x_1} &= \frac{1}{2\sqrt{x_1}} - \lambda = 0 \\ \frac{\partial L}{\partial x_2} &= 1 - 2\lambda = 0 \\ \frac{\partial L}{\partial \lambda} &= 10 - x_1 - 2x_2 = 0 \end{aligned}\right\} \Rightarrow$$

$x_1 = 1, \quad x_2 = \frac{9}{2}$

$\Rightarrow v(p^0, y^0) = \sqrt{1} + \frac{9}{2} = \frac{11}{2}$

$\min_{x_1, x_2} 2x_1 + x_2$

s.t: $\sqrt{x_1} + x_2 = u_0 = \frac{11}{2}$

$L(x_1, x_2, \mu) = 2x_1 + x_2 + \mu(\frac{11}{2} - \sqrt{x_1} - x_2)$

FOCs:
$$\left.\begin{aligned} \frac{\partial L}{\partial x_1} &= 2 - \mu \cdot \frac{1}{2\sqrt{x_1}} = 0 \\ \frac{\partial L}{\partial x_2} &= 1 - \mu = 0 \\ \frac{\partial L}{\partial \mu} &= \frac{11}{2} - \sqrt{x_1} - x_2 = 0 \end{aligned}\right\} \Rightarrow$$

$x_1' = \frac{1}{16}, \quad x_2' = \frac{21}{4}$

$\Rightarrow e(p', u^0) = 2 \cdot \frac{1}{16} + \frac{21}{4} = \frac{43}{8}$

Therefore,

$$I(p', p^0, u^0) \equiv \frac{e(p', u^0)}{e(p^0, u^0)} = \frac{\frac{43}{8}}{10} = \frac{43}{80}$$

(b). In base period,

$\min_{x_1, x_2} 1 \cdot x_1 + 2 \cdot x_2$

s.t: $\sqrt{x_1} + x_2 = u_0$

Set up Lagrangian, and solve for x_1, x_2:

$x_1 = 1, \quad x_2 = u_0 - 1$

$\Rightarrow e(p^0, u^0) = 1 + 2(u_0 - 1) = 2u_0 - 1$

In final period,

$\min_{x_1, x_2} 2 \cdot x_1 + 1 \cdot x_2$

s.t: $\sqrt{x_1} + x_2 = u_0$

Solve this problem, we get:

$x_1' = \frac{1}{16}, \quad x_2' = u_0 - \frac{1}{4}$

$\Rightarrow e(p', u^0) = 2 \cdot \frac{1}{16} + u_0 - \frac{1}{4} = u_0 - \frac{1}{8}$

Hence,

$$I(p', p^0, u^0) \equiv \frac{e(p', u^0)}{e(p^0, u^0)}$$

$$= \frac{u_0 - \frac{1}{8}}{2u_0 - 1}$$

Apparently, $I(p', p^0, u^0)$ varies with u_0.

(c). $\because$ Preference is homothetic

$\therefore$ $e(p, u) = e(p) \cdot u$

(we proved it in Question 4, Pset 4).

$$\therefore I(p', p^0, u^0) \equiv \frac{e(p', u^0)}{e(p^0, u^0)}$$

$$= \frac{e(p') \cdot u^0}{e(p^0) \cdot u^0}$$

$$= \frac{e(p')}{e(p^0)}$$

Therefore, I is independent of u^0.

2.

$\because$ Preference is homothetic

$\therefore$ $x_i(p, y) = x_i(p) \cdot y$

(we proved it in Question 4, Pset 4)

$$\therefore \frac{\partial x_i(p, y)}{\partial y} = x_i(p)$$

By slusky,

$$\frac{\partial x_i(p, y)}{\partial p_j} = \frac{\partial x_i^h(p, u)}{\partial p_j} - \frac{\partial x_i(p, y)}{\partial y} \cdot x_j$$

$$\Rightarrow \frac{\partial x_i(p, y)}{\partial p_j} = \frac{\partial x_i^h(p, u)}{\partial p_j} - x_i(p) \cdot x_j$$

(a). If $\frac{\partial x_i^h}{\partial p_j} > 0$,

since $x_i(p) \cdot x_j > 0$

then $\frac{\partial x_i(p, y)}{\partial p_j} \gtrless 0$

the sign of $\left(\frac{\partial x_i}{\partial p_j}\right)$ is indefinite.

(b). If $\frac{\partial x_i^h}{\partial p_j} < 0$

since $x_i(p) \cdot x_j > 0$

then $\frac{\partial x_i(p, y)}{\partial p_j} < 0$

3. JR 2.17

Assume g is a simple gamble

$$g = (p_1 \circ a_1, p_2 \circ a_2, \cdots p_n \circ a_n)$$

By G3, there exists $\alpha, \beta \in [0, 1]$, such that

$$g \sim (\alpha \circ a_1, (1-\alpha) \circ a_n)$$

$$g \sim (\beta \circ a_1, (1-\beta) \circ a_n)$$

By transitivity,

$$(\alpha \circ a_1, (1-\alpha) \circ a_n) \succsim (\beta \circ a_1, (1-\beta) \circ a_n) \quad (1)$$

$$(\beta \circ a_1, (1-\beta) \circ a_n) \succsim (\alpha \circ a_1, (1-\alpha) \circ a_n) \quad (2)$$

By G4,

$$\left.\begin{array}{l}(1) \text{ holds iff } \alpha \geq \beta \\ (2) \text{ holds iff } \beta \geq \alpha\end{array}\right\} \Rightarrow \alpha = \beta$$

Therefore, α is unique.

4. JR 2.18.

Let $g_1 = (p_1 \circ a_1, \cdots p_n \circ a_n)$

$g_2 = (q_1 \circ a_1, \cdots q_n \circ a_n)$

$g_3 = (r_1 \circ a_1, \cdots r_n \circ a_n)$

g_1, g_2, g_3 are simple gambles.

Construct compound gambles:

$cg_1 = (\alpha \circ g_1, \ (1-\alpha) \circ g_3)$ $\quad \alpha \in [0,1]$

$cg_2 = (\alpha \circ g_2, \ (1-\alpha) \circ g_3)$

$\because$ $g_1 \sim g_2$ by given

$g_3 \sim g_3$

By G5, we have

$cg_1 \sim cg_2$

By G6, we have

$cg_1 \sim \big((\alpha p_1 + (1-\alpha) r_1) \circ a_1, \cdots, (\alpha p_n + (1-\alpha) r_n) \circ a_n\big)$

$cg_2 \sim \big((\alpha q_1 + (1-\alpha) r_1) \circ a_1, \cdots, (\alpha q_n + (1-\alpha) r_n) \circ a_n\big)$

Hence,

$$\big((\alpha p_1 + (1-\alpha) r_1) \circ a_1, \cdots, (\alpha p_n + (1-\alpha) r_n) \circ a_n\big)$$

$$\sim$$

$$\big((\alpha q_1 + (1-\alpha) r_1) \circ a_1, \cdots, (\alpha q_n + (1-\alpha) r_n) \circ a_n\big)$$

5.

Let x_1 = consumption of energy

x_2 = all the other goods

p = price of energy

1 = price of all the other goods.

suppose p has n possible values. i.e

$p = (p_1, p_2, \cdots p_n)$.

with probability distribution

$q = (q_1, q_2, \cdots q_n)$, $\quad q_i \geq 0, \ \sum_{i=1}^{n} q_i = 1$

In state i, price of energy is p_i,

consumers solve the problem:

$$\min_{x_1, x_2} \ p_i \cdot x_1 + 1 \cdot x_2$$

$$s.t: \quad u(x_1, x_2) = u_0$$

Solve the above expenditure minimization problem, we can get $e(p_i, u_0)$.

So the expected expenditure

$$E1 = \sum_{i=1}^{n} q_i \cdot e(p_i, u_0) \longleftarrow$$ expenditure corresponding to randomly fluctuating price

On the other hand, expected price of energy

$$P_E = \sum_{i=1}^{n} q_i \cdot p_i .$$

Under P_E, u_0, consumers solve the problem:

$$\min_{x_1, x_2} \ P_E \cdot x_1 + x_2$$

$$s.t: \quad u(x_1, x_2) = u_0$$

Solving the above problem, we get

$$E_2 = e(P_E, u_0) = e\left(\sum q_i \cdot p_i, \ u_0\right)$$

$\uparrow$ expenditure corresponding to expected price.

Because $e(p, u)$ is concave in p

$$\therefore \ \sum_{i=1}^{n} q_i \, e(p_i, u^0) \leq e\left(\sum_{i=1}^{n} q_i \cdot p_i, \ u_0\right)$$