ECON301a: Advanced Microeconomics
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Problem Set 7: Due Wednesday, 10/20

1. A consumer has an expected utility function given by $u(w) = \log(w)$. He is offered the opportunity to bet on the flip of a coin that has a probability p of coming up heads, (1-p) tails. If he bets x he will have $w + x$ if heads, and $w - x$ if tails. Solve for the optimal x as a function of p. What is the optimal choice of x when $p = 1/2$?

$$U = p \log(w + x) + (1 - p) \log(w - x)$$

$$dU/dx = p/(w + x) - (1 - p)/(w - x) = 0$$

$$p(w - x) = (1 - p)(w + x) = 0$$

Solve for x.

2. A consumer's expected utility function is given by,

$$U(x) = \sum_{i=1}^{n} p_i g(x_i)$$

Show that by transforming $V(x) = AU(x) + B$ you will still get an expected utility function over x_i, and that the new function values a set of gambles (p_i, x_i) in the same way as $U(x)$ did.

This is the relatively easy direction,

$$V(x) = B + \sum_{i=1}^{n} p_i A g(x_i)$$

$$V(x) = \sum_{i=1}^{n} p_i (A g(x_i) + B)$$

Since the linear transform on g() is monotonic. This must be the same ranking.

3. Consider the expected utility given by

$$U = E(g(x)))$$

$g(x) = ax - bx^2$. You are evaluating random additions to wealth $x = w + sz$. $E(z) = \mu$, $E(z^2) = \sigma^2$. You need $g(x)$ to be increasing and strictly concave. If z has a maximum value of z_m what restriction do these restrictions put on the parameters. What is the optimal level of s in terms of the parameters? How is it affected by an increase in σ^2.

$$g'(x) > 0, g''(x) < 0$$

This must hold for all $x < x_m$.

$$E(u) = E(a(w + sz) - b(w + sz)^2)$$

$$E(u) = a(w + s\mu) - b(w^2 + \sigma^2(s)^2 + 2ws\mu)$$

$$0 = a\mu - b(2s\sigma^2 + 2w\mu)$$

$$s = \frac{a\mu - 2w\mu}{b\sigma^2}$$

4. True, False, Uncertain: Three people have initial wealth, W, and are offered the opportunity, at a cost of \$5, to participate in a gamble with pay-offs of \$10 or nothing, with probability 1/2 on each outcome. Suppose individual A accepts the gamble, B rejects it, and C is indifferent. It follows that A has diminishing marginal utility of wealth, B has increasing, and C constant marginal utility of wealth.

   This is simply a test of $u''(W) <> 0$. For C this is zero, risk neutral. For A $u''(W) > 0$ since A is risk loving. For B this is negative since B is risk averse.

5. Consider the "hunter problem" from the book around axioms G3 and G4. Say this hunter prefers a small chance of death, as opposed to sitting at home. Outcomes are given by (1000,10,death). Sitting at home gets you 10 with certainty, and hunting gives a gamble over 1000 (prob = p) and death (prob = (1-p)). The above statement converts to the following formal condition. There is a range of probabilities $0 < p < \bar{p}$ for which the hunter prefers to hunt for all these p's to the option of staying home. Show that such a setup would be inconsistent with the combination of axioms G3 and G4.

   Normally, we would have expected utility falling in the probability of death, $(1-p)$ as this value moves from 0 to 1. It must do this for most values of $(1-p)$. However, our hunter prefers a small prob of death to zero, so this must be increasing over some small range of $(1-p)$. This gives a nonmonotonicity for the prefs in the probability $(1-p)$, and violates axiom G4.