Problem Set 7. Econ301. Fall 2010

1. $g = \{ p \circ (w+x),\ (1-p) \circ (w-x) \}$

$$\max_x u(g) = p\log(w+x) + (1-p)\log(w-x)$$

FOC: $$\frac{\partial u(g)}{\partial x} = \frac{p}{w+x} + \frac{1-p}{w-x} \cdot (-1) = 0$$

$$\therefore \quad x = (2p-1)w$$

when $p = \frac{1}{2}$, $\quad x = 0$.

2. For $\forall g = \{ p_1 \circ g_1, \cdots p_n \circ g_n \}$

where $g_1 \geq g_2 \geq \cdots \geq g_n$

we know $\quad g \sim \{ \alpha \circ g_1,\ (1-\alpha) \circ g_n \}$

where α captures the preference.

An expected utility $u(\cdot)$ satisfies:

$$\frac{u(g_1) - u(g)}{u(g) - u(g_n)} = \frac{1-\alpha}{\alpha}$$

If $\quad V(g) = A \cdot u(g) + B$,

then $$\frac{V(g_1) - V(g)}{V(g) - V(g_n)}$$

$$= \frac{Au(g_1) + B - Au(g) - B}{Au(g) + B - Au(g_n) - B}$$

$$= \frac{u(g_1) - u(g)}{u(g) - u(g_n)}$$

$$= \frac{1-\alpha}{\alpha}$$

Therefore, $u(\cdot)$ and $V(\cdot)$ represent the same preference.

3. $\because$ $g(x)$ is increasing and strictly concave

$$\therefore \quad g'(x) = a - 2bx > 0$$

$$g''(x) = -2b < 0$$

$$\because \quad x = w + s \cdot z, \quad z \leq z_m$$

$$\therefore \quad w + s \cdot z_m < \frac{a}{2b}$$

So the restrictions are:

$$w + s \cdot z_m < \frac{a}{2b}, \quad b > 0$$

$$\max_s E[g(x)] = E[ax - bx^2]$$

$$= E[a(w+sz) - b(w+sz)^2]$$

$$= aw - bw^2 + (as - 2bws) \cdot E(z) - bs^2 E(z^2)$$

$$= aw - bw^2 + (as - 2bws)\mu - bs^2\sigma^2$$

FOC: $$\frac{\partial E[g(x)]}{\partial s} = (a - 2bw)\mu - 2bs\sigma^2 = 0$$

$$\therefore \quad s = \frac{(a-2bw)\mu}{2b\sigma^2}$$

If $\sigma^2 \uparrow$, then $s \downarrow$.

4. $g = \{ \frac{1}{2} \circ (w+5), \frac{1}{2} \circ (w-5) \}$

$u(g) = \frac{1}{2} u(w+5) + \frac{1}{2} u(w-5)$

$\because$ A accepts the gamble

$\therefore$ $u(g) > u(w)$

$\therefore$ $\frac{1}{2} u(w+5) + \frac{1}{2} u(w-5) > u(w)$

Apply Taylor expansion, we have:

$u''(w) > 0$

$\therefore$ For A, marginal utility of wealth is increasing.

Similarly, B has decreasing, and C has constant marginal utility of wealth.

5. Rank 3 outcomes:

$1000 \succ 10 \succ$ death.

Let $g_n = \{ p \circ 1000, (1-p) \circ \text{death} \}$

By G3, $\exists \alpha \in [0,1]$, such that

$\{ \alpha \circ 1000, (1-\alpha) \circ \text{death} \} \sim 10$ $(*)$

By the statement,

$\forall p \in (0, \bar{p})$,

$\{ p \circ 1000, (1-p) \circ \text{death} \} \succ 10$ $(**)$

$(*) + (**)$:

$\therefore \{ p \circ 1000, (1-p) \circ \text{death} \} \succ \{ \alpha \circ 1000, (1-\alpha) \circ \text{death} \}$

By G4, it must be:

$p > \alpha$

$\therefore$ $\alpha < p$, for $\forall p \in (0, \bar{p})$

$\therefore$ $\alpha = 0$

Then $(*)$ becomes:

death ~ 10

which is clearly an contradition.