Problem Set 8 Econ301.

1. $g_1 = \{0.5 \circ 990,\ 0.5 \circ 1011\}$

$u(g_1) - u(1000)$

$= \frac{1}{2}\cdot\frac{990^{1-\sigma}}{1-\sigma} + \frac{1}{2}\cdot\frac{1011^{1-\sigma}}{1-\sigma} - \frac{1000^{1-\sigma}}{1-\sigma}$

The consumer will accept g_1 if $u(g_1) > u(1000)$, turn it down if $u(g_1) < u(1000)$.

By calculation, when $\sigma = 0, 1 \cdots 9$, $u(g_1) > u(1000)$; when $\sigma = 10, 11 \cdots$ $u(g_1) < u(1000)$.

Set $\sigma = 10$

$g_2 = \{0.5 \circ 900,\ 0.5 \circ (1000+y)\}$

The consumer will accept g_2 iff $u(g_2) \geq u(1000)$.

$\Leftrightarrow \frac{1}{2}\cdot\frac{900^{1-\sigma}}{1-\sigma} + \frac{1}{2}\cdot\frac{(1000+y)^{1-\sigma}}{1-\sigma} \geq \frac{1000^{1-\sigma}}{1-\sigma}$

$\Rightarrow y \geq -2062.15$, a negative number

$g_3 = \{0.5 \circ 925,\ 0.5 \circ (1000+y')\}$

The consumer will accept g_3 iff $u(g_3) \geq u(1000)$

$\Rightarrow y' \geq -2571.71$, a negative one as well.

In both cases, the results are odd when $\sigma = 10$.

2. Gain from illegal parking is $(w+G) + \tilde{z}$

where $\tilde{z} = 0$ with probability $(1-p)$, $\tilde{z} = -F$ with probability p.

$E[u(w+G+\tilde{z})]$

$= E[u(w+G) + u'(w+G)\cdot\tilde{z} + \frac{1}{2}u''(w+G)\cdot\tilde{z}^2]$

$= u(w+G) + u'(w+G)E(\tilde{z}) + \frac{1}{2}u''(w+G)E(\tilde{z}^2)$

$\because$ $p \cdot F = k$, a constant

$\therefore$ $E(\tilde{z}) = -F \cdot p = -k$

$E(\tilde{z}^2) = F^2 \cdot p = \frac{k^2}{p}$

$\therefore$ $E[u(w+G+\tilde{z})]$

$= u(w+G) - u'(w+G) + \frac{1}{2}u''(w+G)\cdot\frac{k^2}{p}$

$\because$ $u''(\cdot) < 0$

$\therefore$ as $p\uparrow$, $E[u(w+G+\tilde{z})]\uparrow$

Therefore, increasing p will raise the expected utility of illegal parking, and cause more illegal parking.

3.

	Red	Blue	Yellow
A	w	0	0
B	0	w	0
C	w	0	w
D	0	w	w

For A: $P_R \cdot u(w) + P_B \cdot u(0) + P_Y \cdot u(0) = \frac{1}{3}w$

For B: $P_R \cdot u(0) + P_B \cdot u(w) + P_Y \cdot u(0) \in [0, \frac{2}{3}w]$

$\Rightarrow \quad u(A) = \frac{1}{3}w, \quad u(B) = \min[0, \frac{2}{3}w] = 0$

$\Rightarrow \quad \max_{A,B} \; u(A), u(B) = \max \; \frac{1}{3}w, 0 = \frac{1}{3}w$

$\Rightarrow$ The player will choose A over B.

For C: $P_R \cdot u(w) + P_B \cdot u(0) + P_Y \cdot u(w) \in [\frac{1}{3}w, \frac{2}{3}w]$

For D: $P_R \cdot u(0) + P_B \cdot u(w) + P_Y \cdot u(w) = \frac{2}{3}w$

$\Rightarrow \quad u(C) = \min[\frac{1}{3}w, \frac{2}{3}w] = \frac{1}{3}w$

$\quad\;\; u(D) = \frac{2}{3}w$

$\Rightarrow \quad \max_{C,D} \; u(C), u(D) = \max \; \frac{2}{3}w, \frac{1}{3}w = \frac{2}{3}w$

$\Rightarrow$ The player will choose D over C.

Hence, the Ellsberg paradox choice can be exhibited by max-min behavior.

4. $\max \; u(c_1) + \beta q u(c_{2,1}) + \beta(1-q) u(c_{2,2})$

s.t.: $c_1 + p_1 \cdot c_{2,1} + p_2 \cdot c_{2,2} = y$

(a). $L(c_1, c_{2,1}, c_{2,2}) = u(c_1) + \beta q u(c_{2,1}) + \beta(1-q) u(c_{2,2})$
$\quad + \lambda(y - c_1 - p_1 \cdot c_{2,1} - p_2 \cdot c_{2,2})$

FOCs: $\frac{\partial L}{\partial c_1} = u'(c_1) - \lambda = 0$

$\frac{\partial L}{\partial c_{2,1}} = \beta q u'(c_{2,1}) - \lambda p_1 = 0$

$\frac{\partial L}{\partial c_{2,2}} = \beta(1-q) \cdot u'(c_{2,2}) - \lambda p_2 = 0$

(b). Assume $u(c) = \log c$

then FOCs become:

$\frac{1}{c_1} - \lambda = 0$

$\frac{\beta q}{c_{2,1}} - \lambda p_1 = 0$

$\frac{\beta(1-q)}{c_{2,2}} - \lambda p_2 = 0$

$\Rightarrow \quad \frac{c_{2,1}}{c_{2,2}} = \frac{p_2}{p_1} \cdot \frac{q}{1-q}$, does not depend on β.

(c). From (b), we have:

$\frac{c_1}{c_{2,1}} = \frac{p_1}{\beta \cdot q}$

$\frac{c_1}{c_{2,2}} = \frac{p_2}{\beta \cdot (1-q)}$

If all consumers have same β and q, then $\frac{c_1}{c_{2,1}}$ and $\frac{c_1}{c_{2,2}}$ will be the same across all consumers.