ECON301a: Advanced Microeconomics
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Problem Set 9: Due Wednesday, 11/17

1. True, False, Uncertain: Explain
   (a) All global increasing returns to scale production functions are homothetic.
   (b) All global constant returns to scale production functions are homothetic.
   (c) All homothetic production functions are either globally increasing, decreasing, or constant returns to scale.
   (d) For any homothetic production technology the ratio in which factors are used will not change as output expands holding factor prices constant.

2. Sketch the isoquants of production for a Leontief production function which is given by,

$$y = \min(ax_1, bx_2).$$

   Describe why the elasticity of substitution (when defined) is zero.

3. Derive a Le Chatelier principle for conditional factor demands. In other words using features comparing the short run cost function, and the long run cost function show that ,

$$\frac{\partial x_i^{LR}(w,y)}{\partial w_i} \geq \frac{\partial x_i^{SR}(w,y)}{\partial w_i}$$

4. Derive the profit function, $\pi(p,w)$ for the Cobb-Douglas production technology,

$$y = x_1^{\alpha} x_2^{\beta}.$$

   What restrictions do you need to put on α and β to make sure this function is well defined.

5. What is the cost function for the linear production technology,

$$y = \sum_{i=1}^{n} a_i x_i$$

6. A real function is called "superadditive" if

$$f(z^1 + z^2) \geq f(z^1) + f(z^2)$$

   Show that every cost function is super additive in input prices. Use this to prove that the cost function is nondecreasing in input prices without requiring it to be differentiable.