Notes on risk aversion (Pratt, Econometrica 1964)

These notes outline some of the usual metrics for thinking about risk aversion. Assume that a consumer faces a “fair gamble”, $z$.

$$E(z) = 0, \quad E(z^2) = \sigma^2$$

Find the payoff that would make the consumer indifferent between taking the gamble or not.

$$u(c - \pi) = Eu(c + z)$$

This is a “local measure” which means that a Taylor series will suffice.

$$u(c) - u'(c)\pi \approx E(u(c) + cuz(c) + (1/2)z^2u''(c))$$

$$-u'(c)\pi \approx (1/2)\sigma^2u''(c)$$

$$\pi(c) \approx (1/2)\sigma^2 \frac{-u''(c)}{u'(c)}$$

Note: this depends on $c$.

The value, $-\frac{u''(c)}{u'(c)}$, is defined as the coefficient of absolute risk aversion.

Relative risk aversion asks the same question in proportion to current consumption.

$$u(c - \pi^*c) = Eu(c + zc)$$

$$\pi^*(c) \approx (1/2)\sigma^2 \frac{-u''(c)c}{u'(c)}$$

The value, $-\frac{u''(c)c}{u'(c)}$, is the coefficient of relative risk aversion.

<table>
<thead>
<tr>
<th>Utility</th>
<th>Absolute RA</th>
<th>Relative RA.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u(c)$</td>
<td>$-\frac{u''(c)}{u'(c)}$</td>
<td>$-\frac{u''(c)c}{u'(c)}$</td>
</tr>
<tr>
<td>$A + Bc$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\frac{1}{\gamma}e^{-\gamma c}$</td>
<td>$\gamma$</td>
<td>$\gamma c$</td>
</tr>
<tr>
<td>$\log c$</td>
<td>$\frac{1}{c}$</td>
<td>1</td>
</tr>
<tr>
<td>$\frac{1}{1-\gamma}e^{1-\gamma}$</td>
<td>$\gamma$</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>$Ac + Bc^2$</td>
<td>$-\frac{2Bc}{A+2Bc}$</td>
<td>$-\frac{2Bc}{A+2Bc}$</td>
</tr>
<tr>
<td>$\frac{1}{1-\gamma}(c + \eta)^{1-\gamma}$</td>
<td>$\frac{1}{c+\eta}$</td>
<td>$\frac{1}{1+(\eta/c)}$</td>
</tr>
</tbody>
</table>
Utility example:

\[ u(c) = \frac{1}{1 - \gamma} c^{1-\gamma} \]

\[ c = 100 \]

Sample equal plus or minus 10 percent with probability 1/2 each.
In other words, \( z = 0.1, c = 110 \) with probability 1/2 and \( z = -0.1, c = 90 \) with probability 1/2

\[ \sigma^2 = (1/2)(0.1)^2 + (1/2)(0.1)^2 = 0.01 \]

Remember,

\[ u(c - \pi^* c) = Eu(c + zc) \]

\[ \pi = (1/2)\sigma^2 \gamma \]

Table 2: RA’s for various utility functions

<table>
<thead>
<tr>
<th>( \gamma )</th>
<th>( \pi^* = (1/2)\sigma^2 \gamma )</th>
<th>( c - \pi^* c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.01</td>
<td>99</td>
</tr>
<tr>
<td>5</td>
<td>0.025</td>
<td>97.5</td>
</tr>
<tr>
<td>10</td>
<td>0.05</td>
<td>95</td>
</tr>
<tr>
<td>20*</td>
<td>0.10</td>
<td>93</td>
</tr>
<tr>
<td>50*</td>
<td>0.25</td>
<td>91.5</td>
</tr>
</tbody>
</table>

*Note: These last two are done numerically, by simulating the utility function, and then finding the certainty consumption that gives the same utility as the expected utility. If one used the Taylor approximation, then the value in both cases is clearly wrong since the certainty consumption is less than or equal to the minimum gamble result.