Fin285a: Computer Simulations and Risk Assessment
Section 9.1-9.2
Options and Partial Risk Hedges
Reading: Hilpisch, 290-294
Option valuation: Analytic

- Black/Scholes function

Option valuation: Monte-carlo

Partial hedging example I: Single domestic equity

Partial hedging example II: International equity
European call option

- $P_t$ stock price
- $S$ strike price
- $h$ periods to expiration
- Option to purchase stock at price $S$, time $t+h$

\[ V_{t+h} = \max(P_{t+h} - S, 0) \]  \hspace{1cm} (9.1.1)

\[ V_t = F_C(P_t, S, \sigma, r_f, h) \]  \hspace{1cm} (9.1.2)
European put option

- $P_t$ stock price
- $S$ strike price
- $h$ periods to expiration
- Option to sell stock at price $S$, time $t + h$

\[ V_{t+h} = \max(S - P_{t+h}, 0) \] (9.1.3)

\[ V_t = F_P(P_t, S, \sigma, r_f, h) \] (9.1.4)
Black/Scholes formula

- Stock price follows geometric Brownian motion
  \[ \log(P_t) = \log(P_{t-1}) + z_t \]
  \[ z_t \sim N(\mu, \sigma) \]
- Constant variance
- Python function `callput.py`
- No early exercise
- See next slide
def callput(price, strike, vol, rf, tmat):
    # usage: pcall, pput, delta =
    #     callput(price, strike, vol, rf, tmat);
    # Standard Black-Scholes values for plain vanilla
    #     European options
    # price = current price
    # strike = strike price
    # vol = volatility (in std/year)
    # rf = risk free interest rate (annual rate)
    # tmat = maturity in fractional years
    # returns:
    #     call valuation
    #     put valuation
    #     delta hedge ratio (dval/dp)
Option valuation: Monte-carlo

Partial hedging example I: Single domestic equity

Partial hedging example II: International equity

Option valuation: Analytic
Risk neutral simulation

- Simulate Brownian motion for stock price
- Special parameters
  - Drift (mean) = risk free rate (adjusted)
  - Volatility equal true volatility
- Option value = discounted payoff of the option
- Works for many types of options
- Heavily used model
Reminder on risk neutral rates

- Continuous compounding
- $R_f$ is the interest rate
- $r_f$ is continuous compound interest (per period $t$)
  \[ r_f = \log(1 + R_f) \]
- Funds grow (future value):
  \[ B_t = B_0 e^{r_f t} \]
- Discounting (present value):
  \[ C_t / e^{r_f t} = C_t e^{-r_f t} \]
- This is standard in any options/derivatives course
Expectations for log normals

\[
\log(Y) \sim N(\mu, \sigma^2) \quad (9.1.5)
\]

\[
E(e^Y) \neq e^\mu \quad (9.1.6)
\]

\[
E(e^Y) = e^{\mu + (1/2)\sigma^2} \quad (9.1.7)
\]

\[
r_t = \log(1 + R_t) \quad (9.1.8)
\]

\[
E(R_t) = E(e^{r_t}) - 1 \\
= e^{E(r_t) + (1/2)\text{var}(r_t)} - 1 \quad (9.1.10)
\]
Risk neutral drift

- Simulate Brownian motion for stock price
- Use special adjusted drift (over h periods)

\[ r_t = \log(P_{t+1}) - \log(P_t) \]

\[ \mu = E(r_t) = r_f - (1/2)\sigma^2 \]

\[ \text{var}(r_t) = \sigma^2 \]

\[ E(R_t) = E(e^{r_t}) - 1 = e^{r_f - (1/2)\sigma^2 + (1/2)\sigma^2} - 1 \]
\[ = e^{r_f} - 1 = R_f \]

- So the expected arithmetic return is the risk free rate
Implementation: European call option

- Draw returns log normally
  \[ r_t = N(\mu, \sigma^2) \]
  \[ P_{t+h} = P_t e^{(\sum_{j=1}^{h} r_{t+j})} \]

- Option value = discounted payoff of the option
  \[ \text{Call}: e^{-rfh} E(\max(P_{t+h} - S, 0)) \]
  \[ S = \text{strike} \]
  \[ h = \text{periods to expiration} \]

- We will see a Python example soon

- Note: options with early exercise are much trickier
Partial hedging example I: Single domestic equity

Partial hedging example II: International equity
Simple option example

- 100 shares of S&P500
- Current price = 1
- Cover 75 shares with put options
  - European
  - Expiration in 1 day
- Strike price = 1 (at the money)
- Python: putSP.py
Valuation today (marking to market)

• Option value today
  ⇒ Use Black/Scholes formula
  ⇒ Often you would put in actual market price here

• putSP.py also gives monte-carlo example
Put option valuation tomorrow

- Option expires
- Price increase: zero
- Price decrease
  - $\Rightarrow$ strike - Price(tomorrow)
  - $\Rightarrow$ 1 - Price(tomorrow)
Python example

- Very nonnormal distribution
- Not an easy analytic distribution
- Importance of computer simulation for VaR
- Can easily change hedging amount
Partial hedging example II: International equity
International option portfolio

• **optdist.py**
• Portfolio with US/HK (SP500 and MSCI Hong Kong ETF (EWH))
• Start price equals last price from data files
• $100 position, initially split 50/50
• Hedge US only
  ⇨ 20 day at the money put option
  ⇨ Price from Black/Scholes (Python function)
  ⇨ VaR in 20 days (expiration)

• Bootstrap: independent over time, but not across countries
  ⇨ This is tricky

• Distribution is non normal (skewed)
• Compare to normal approximation for VaR
Option calculations

- $s_x = \text{shares (or options)}$
- $P_{\text{Strike}} = \text{strike price}$

$s_{\text{put}} = s_{\text{US}}$  \hspace{1cm} (9.1.11)

$P_{\text{Strike}} = P_{\text{US},t}$  \hspace{1cm} (9.1.12)

$V_t = P_{\text{US},t}s_{\text{US}} + P_{\text{HK},t}s_{\text{HK}} + P_{\text{put},t}s_{\text{put}}$  \hspace{1cm} (9.1.13)

$V_{t+20} = P_{\text{US},t+20}s_{\text{US}} + P_{\text{HK},t+20}s_{\text{HK}} +$ \hspace{1cm} (9.1.14)

$\max(P_{\text{Strike}} - P_{\text{US},t+20}, 0)s_{\text{put}}$  \hspace{1cm} (9.1.15)

$\text{VaR}(p) = -(q_p(V_{t+20}) - V_t)$  \hspace{1cm} (9.1.16)
Summary

• It is easy to get non normal distributions with options
• These are complex and often require simulations to correctly estimate risk
• Often more dramatic than standard symmetric fat tail distributions