Exploring Financial Instability Through Agent-based Modeling
Part 2: Time Series, Adaptation, and Survival

Blake LeBaron
International Business School
Brandeis University

Mini course
CIGI-INET: False Dichotomies
Where are we going?

- Part 1:
  - What are agent-based models?
  - Simple models from finance
- Part 2:
  - Adaptation and time series
  - Heterogeneous gain learning
- Part 3:
  - Current directions in agent design and applications
  - Empirical validation
  - Instability and macro connections
Overview

Learning and Time Series
Model Structure
Basic Simulation Plots
Large Swings From Fundamentals
Momentum, Risk, and Return
Consumption
Summary
Learning and Time Series

Model Structure

Basic Simulation Plots

Large Swings From Fundamentals

Momentum, Risk, and Return

Consumption

Summary
Time series and populations

Time series features

Strategy populations
Financial empirical summary

- **Short term**
  - Volatility persistence
  - Leptokurtic (fat tailed) return distributions
  - Uncorrelated returns

- **Long term**
  - Volatility persistence
  - Return predictability fundamental mean reversion
  - Momentum
  - Risk and return relationships
  - Consumption and returns
Features and dynamics

We will go through these time series features, demonstrating how they are connected to underlying learning dynamics.
Models generating long range features

▷ Boswijk et al. (2007)
  • Simple, few type
  • Estimation
  • Long horizon only
▷ LeBaron (2013 forthcoming)
  • More complicated
  • All horizons
  • Richer evolutionary story
Readings


Model Structure
Key features

▷ Core strategies
  • Adaptive and fundamental types
▷ Data perspectives (gain levels)
  • Short and long term views of history
  • Learning (or not learning) stationarity
▷ Learning about return **and** risk
▷ Trader/strategy survival
Connections to other economic ideas

- Limits to arbitrage
- Noise trader risk
- Minsky instability
Model components

1. Economy (simple)
2. Agents (risk/return)
3. Forecasting rules
4. Trading
5. Evolution
Economy

▶ Stock
  - Stochastic dividend
  - Random walk (growth and volatility calibrated)
  - Finite supply
  - Endogenous price, $P_t$

▶ Risk free
  - $R_f = 0$
  - Infinite supply
Agents: portfolio choice

\[ E_{i,t}(R_{t+1}) = \text{Expected return} \]
\[ \hat{\sigma}_{i,t+1}^2 = \text{Expected variance} \]

\[ \alpha_{i,t} = f(E_{i,t}(R_{t+1}), \hat{\sigma}_{i,t+1}^2) \]
\[ = \text{Fraction of savings in stock} \]

\[ \alpha_{i,t} = \frac{E_{i,t}(R_{t+1}) - R_f}{\gamma \hat{\sigma}_{i,t+1}^2} \]
Agents: portfolio choice

\[ S_{i,t} = \frac{\alpha_{i,t}(1 - \lambda)W_{i,t}}{P_t} = \text{Shares} \]

\[ B_{i,t} = (1 - \alpha_{i,t})(1 - \lambda)W_{i,t} = \text{Cash} \]

\[ C_{i,t} = \lambda W_{i,t} = \text{Consumption} \]
Forecast rules

1. Adaptive expectations (technical/trend following)
2. Mean reverting (fundamental)
3. Noise (short term forecasts)
4. Buy and hold (long range)
5. Variance forecasts
Adaptive expectations

\[ f_{t+1,j} = (1 - g_j) f_{t,j} + (g_j) R_t \]

\[ g_i = \text{Gain} \]
Mean reverting (fundamental)

\[ pd_t = \log \left( \frac{P_t}{D_t} \right) \]

\[ f_{t+1,j} = \bar{R} + \beta_{t,j} (pd_t - \bar{p}d) \]

▷ Estimation: Recursive least squares
▷ Constant gain, \( g_j \)
Other rules

➢ Noise
  ● Short range linear forecasts
  ● Recursive least squares

➢ Buy and hold
  ● Long range expected returns and variance
  ● Relatively passive
Variance forecasts

$$\epsilon_{t,j} = (R_t - f_{t,j})$$

$$\hat{\sigma}^2_{t+t,j} = (1 - g_j^\sigma)\hat{\sigma}^2_{t,j} + g_j^\sigma \epsilon^2_{t,j}$$

- Adaptive
- Riskmetrics/GARCH(1,1)
- Similar across agents
- Differ in gain $g_j^\sigma$ levels
- Gain:
  - Horizon/memory
  - Signal/noise
Evolution: rule selection

Agents → Rules
Gain range

- Discrete gain levels
- Half life experiments
  - All gain: [50, 18, 7, 2.5, 1] half-lives in years
  - Low gain: 50 years only
  - High gain: 1 – 5 years only
\[ \hat{y}_t = \sum_{j=1}^{m} (1 - g)^j y_{t-j} \]
Forecast state space (by gain)

- **Forecast** ($g_j$)
  - Adaptive: $5 \times 5$
  - Fundamental: $5 \times 5$

- **Variance** ($g_j^\sigma$)
  - Adaptive: $5 \times 5$

- **Noise**
  - $5 \times 5$

- **Buy and Hold**
  - $1 \times 1$
Forecast state space (by gain)

Adaptive

Variance($g_j^\sigma$)

5 x 5

5 x 5

Agent

Forecast($g_j$)

5 x 5

Fundamental

5 x 5

Noise

5 x 5

Buy and Hold

1 x 1
\[ z_i(P_t) = \frac{a_i(P_t)(1 - \lambda)W_{t,i}}{P_t} \]

\[ \sum_{i=1}^{I} z_i(P_t) = 1 \]
### Brief parameter table

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$ (CRRA)</td>
<td>3.5</td>
</tr>
<tr>
<td>$(W/C)(\lambda)$</td>
<td>40</td>
</tr>
<tr>
<td>Agents</td>
<td>16000</td>
</tr>
<tr>
<td>Rules</td>
<td>4000</td>
</tr>
<tr>
<td>$\mu_D$</td>
<td>0.02</td>
</tr>
<tr>
<td>$\sigma_D$</td>
<td>0.07</td>
</tr>
<tr>
<td>Gain values</td>
<td>$[50, 18, 7, 2.5, 1]$</td>
</tr>
</tbody>
</table>

Note: Gain values are in annualized half lives.
Basic Simulation Plots
Simulation summary

![Simulation graphs showing Price, Weekly returns, and Trading Volume over years.](image-url)
Time series features

- Asymmetry in up and down markets
- Volatility/volume increases in falling markets
Data sets

▷ Sources
  ● CRSP-VW (1926-2011) (monthly/daily)
  ● Shiller (1871-1925)
  ● Schwert (1886-1925) (daily returns)
  ● “Measuring worth” (risk free 1871-1925)

▷ Series used
  ● Returns, excess returns
  ● Price/dividend (P/D) ratios
  ● Monthly realized volatility (daily returns)

▷ Simulations
  ● 100,000 week burn in
  ● 100,000 week sample (≈ 1900 years)
S&P Price/dividend ratio

The graph shows the S&P Price/dividend ratio from 1920 to 2010. The ratio generally increases over time, with significant increases around the 1990s and 2000s.
S&P Price/earnings ratio
Weekly return distributions
Weekly return autocorrelations

![Graph showing weekly return autocorrelations with two lines: one for Simulation and another for CRSP-VW. The x-axis represents the autocorrelation lag in weeks, ranging from 0 to 25, while the y-axis represents the return autocorrelation, ranging from -0.04 to 0.10. The graph displays fluctuations in autocorrelation values at different lags.]
Monthly volatility autocorrelation

![Graph showing monthly volatility autocorrelation with two lines: U.S. merged and Simulation. The x-axis represents lag in months ranging from 0 to 25, and the y-axis represents autocorrelation ranging from 0 to 0.7. The graph shows a declining trend in autocorrelation with increasing lag.]
Annual return summary statistics

Excess Returns

\[ R_{e,t} = R_t - R_{f,t} \]

<table>
<thead>
<tr>
<th>Series</th>
<th>Merged U.S. 1872-2011</th>
<th>Simulation 1900 years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean excess return</td>
<td>7.55</td>
<td>8.47</td>
</tr>
<tr>
<td>Std.</td>
<td>21.25</td>
<td>20.82</td>
</tr>
<tr>
<td>Autocorrelation</td>
<td>-0.09</td>
<td>-0.09</td>
</tr>
</tbody>
</table>
Large Swings From Fundamentals

Momentum, Risk, and Return

Consumption

Summary
Simulation price/dividend ratios

- P/D ratio over years
- Weekly returns over years
- Trading volume over years
Long range return forecasts

\[ R_{t+1} - R_{f,t+1} = \alpha + \beta \log(P_t/D_t) \]

<table>
<thead>
<tr>
<th>Series</th>
<th>( \beta )</th>
<th>R-squared</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRSP (quarterly)</td>
<td>-0.032</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td></td>
</tr>
<tr>
<td>CRSP (annual)</td>
<td>-0.127</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>(0.015)</td>
<td></td>
</tr>
<tr>
<td>Simulation (quarterly)</td>
<td>-0.119</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td></td>
</tr>
<tr>
<td>Simulation (annual)</td>
<td>-0.464</td>
<td>0.33</td>
</tr>
<tr>
<td></td>
<td>(0.015)</td>
<td></td>
</tr>
</tbody>
</table>

See: Lettau and Ludvigson (2010), Cochrane (2011)
Readings


CRSP P/D and volatility $\sigma_{x,y} = -0.22$
Long range volatility forecasts

\[
\log(\sigma_{t+1}^2) = \alpha + \beta_1 \log(P_t/D_t) + \beta_2 \log(\sigma_t^2)
\]

<table>
<thead>
<tr>
<th>Series</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>R-squared</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P 26-1993</td>
<td>-0.81</td>
<td></td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>(0.13)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.18</td>
<td>0.70</td>
<td>0.51</td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
<td>(0.03)</td>
<td></td>
</tr>
<tr>
<td>Simulation</td>
<td>-2.83</td>
<td></td>
<td>0.48</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.57</td>
<td>0.71</td>
<td>0.68</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.01)</td>
<td></td>
</tr>
</tbody>
</table>
Strategy forecasts

![Graphs showing price, weekly returns, and return forecast over years. The graphs compare Adaptive and Fundamental strategies.](image-url)
Population dynamics

Wealth shift = 4 % / year
Fundamental strategy by var gain
Wealth across forecast gains

Adaptive

Fundamental

Noise
Wealth across variance gains

Adaptive

Fundamental

Noise
Explanations for swings

- Limits to arbitrage
  - Not enough wealth in stabilizing strategies
  - Fundamental strategies don’t perform well in risk/return space
- Some fundamental strategies get scared (also too aggressive in rising markets)
- Risk perceptions attenuate fundamental trading
Explanation for downside instability

Aggregate share demand:

\[ S_t = \sum_{i=1}^{I} \alpha_i(P_t)(S_{t-1,i}(P_t + D_t) + B_{t-1,i}) \]

Which can be split into two parts,

\[ S_t = \sum_{i=1}^{I} \alpha_i(P_t)S_{t-1,i} + \sum_{i=1}^{I} \alpha_i(P_t)\left(\frac{S_{t-1,i}D_t + B_{t-1,i}}{P_t}\right) \]
Who determines prices?

- Marginal trader
- Many agents are inframarginal
- Prices not average of all beliefs
- Can swing quickly
- Some trading mechanical (rebalancing)
Momentum, Risk, and Return
Conditional returns

▷ Empirical methodology:
  ● Kernel regression
  ● Bandwidth: Randomized cross validation on simulation series
  ● Structure exploration (shared nonlinear features)

▷ Relationships
  ● Momentum forecasts
    ◇ Lagged 6 months, 1 month wait, forecast 6 months
    ◇ \( (6 - 1 - 6) \)
  ● Current volatility and future excess returns (risk/return)
(6,1,6) Month Strategy

![Graph showing the relationship between lagged 6-month return and future 6-month return for the U.S. merged and simulation data.](image_url)
Dow firm momentum 25/50/75 quantiles

Lagged (6 month) return vs. Expected future (6 month + 1) return
Current volatility and future returns

![Graph showing the relationship between lagged variance and future excess return.](image)

- **Lagged variance (quarter)**
- **Future excess return (quarter)**

- **Red line**: U.S. merged
- **Blue line**: Simulation

LeBaron
Commentary

- Diffusion of beliefs through the population
  - Information processing moving across gains
- Transition from momentum to mean reversion
- Asymmetric price dynamics (rise slowly/fall fast)
Consumption
$C_{i,t} = \lambda \hat{W}_{i,t}$

$\hat{W}_{i,t} = \sum_{k=1}^{K} \omega^k W_{i,t-k}$

<table>
<thead>
<tr>
<th>Series</th>
<th>$E\Delta(\log(C_t))$ Annual %</th>
<th>$\sigma(\Delta \log(C_t))$ Annual %</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. (1947-2010)</td>
<td>3.33</td>
<td>1.72</td>
</tr>
<tr>
<td>$C_{i,t} = \lambda W_{i,t}$</td>
<td>1.77</td>
<td>13.24</td>
</tr>
<tr>
<td>$C_{i,t} = \lambda \hat{W}_{i,t}$</td>
<td>1.67</td>
<td>1.59</td>
</tr>
</tbody>
</table>
Summary
Common to most agent-based markets

- Instability (experiments)
- Short run
  - Fat tail return distributions
  - Volatility persistence
- Mechanisms
  - Interactions between adaptive and fundamental types
  - Wealth adjusting to successful strategies
  - Leverage not necessary
Features of this market

- Survival of multiple gain levels
- Importance of risk in strategy fitness
- Calibrated (small) aggregate shocks
- Small shifts in wealth across strategies
- Interpretable strategies (basis?)
- Fast versus slow learning
  
Interesting counterfactuals

- Passive wealth (too large?)
- Magnitude of predictability/mean reversion
- Strong counter cyclical volatility
- Consumption
- Interest rates?
Volatility and risk perceptions
1. Some traders overreact to recent volatility
2. Discount $P_t$, and increases forecast heterogeneity
Volatility forecasts take time to sweep through heterogeneous beliefs
Key mistake: Short term volatility perspective
Changing risk perception versus risk aversion
Summary

- Model structure
- Statistical features
  - Short horizon
  - Long horizon
- Evolutionary questions
  - Why can’t stabilizing strategies gain more wealth?
  - Why can’t long memory strategies gain more wealth?
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