Wealth Evolution and Financial Instability

Blake LeBaron
International Business School
Brandeis University
Overview

- Why another agent-based market?
- New model description
- Empirical summary
- Crash dynamics
- Summary
Why Another Agent-based Market?

➡ Simple market weaknesses (few type)
  • Dynamics replication
  • Fixed strategy sets

➡ Complicated market weaknesses
  • Difficult to analyze
  • Continually changing strategy sets
Modeling Strategy

➡ Operate with just enough complexity for:

• Reliable empirical replications
• Rich evolutionary dynamics
• Time series and behavior coevolution
Overview

- Types of agent-based financial markets
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- Summary and future
Model Structure

- Time period = One week
- Securities
- Agents
- Forecasts
- Market clearing/price determination
Securities

➡ Risky asset
  • Stochastic dividend (random walk)
  • U.S. data: growth and volatility
  • Fixed supply = One share

➡ Risk free asset
  • Infinite supply
  • Zero interest
Agents

- Utility maximizing
- Constant relative risk aversion
- No leverage, no short sales
- Consume constant fraction of wealth
Agent Portfolio Choice (CRRA)

\[ \alpha_{t,i} \left( r_{t+1} \right) - r_f + \frac{(1/2)\sigma_{t,i}^2}{\gamma\sigma_{t,i}^2} \]

\[ \alpha_{t,i} = \text{Fraction of wealth in risky asset} \]

\[ E_{t,i} \left( r_{t+1} \right) = \text{Forecast expected return (logs)} \]

\[ \sigma_{t,i}^2 = \text{Forecast variance} \]

\[ \gamma = \text{Relative risk aversion} = 3.5 \]
Wealth and Consumption

\[ W_{t,i} = (P_t + D_t)S_{t-1,i} + B_{t-1,i}(1 + r_f) \]

\[ W_{t,i} = P_tS_{t,i} + B_{t,i} + C_{t,i} \]

\[ C_{t,i} = \lambda W_{t,i} \]

\[ \lambda = 0.0004 \]

\[ r_f = 0 \]

\[ P_tS_{t,i} + B_{t,i} = (1 - \lambda)((P_t + D_t)S_{t-1,i} + B_{t-1,i}(1 + r_f)) \]
Forecast Families

1. Trend: Adaptive expectations
2. Fundamental: Price/dividend regressions
3. Noise trader/short term regressions
4. Buy and hold (constant fraction)
Long Range Forecasts

\[ r_t = \log\left(\frac{P_t + D_t}{P_{t-1}}\right) \]

\[ E_t(r_{t+1}) = \bar{r}_t \]

\[ E_t(\sigma_t^2) = \bar{\sigma}_t^2 \]

\[ (p / d)_t = \log\left(\frac{P_t}{D_t}\right) \]

\[ E_t((p / d)_{t+1}) = \overline{p / d}_t \]
Adaptive Expectations

\[ E_{t,j}(r_{t+1}) = \bar{r}_t + m(E_{t-1,j}(r_t) - \bar{r}_t) + g_j(r_t - E_{t-1,j}(r_t)) \]
Price/Dividend Regression

\[ E_{t,j}(r_{t+1}) = \bar{r}_t + \beta_{t,j}(p_t / d_t - \bar{p} / \bar{d}_t) \]

Project onto mean over future 52 weeks
Noise Trader: AR regression

\[ E_{t,j}(r_{t+1}) = \bar{r}_t + \beta_{t,j}(r_t - \bar{r}_t) \]
Buy and Hold: Constant Fraction

\[ E_{t,j}(r_{t+1}) = \bar{r}_t \]
\[ E_{t,j}(\sigma_{t+1}^2) = \bar{\sigma}_t^2 \]
Recursive Least Squares

\[ \beta_{t,j} = \text{Linear forecast coefficient} \]
\[ \beta_{t,j} = h(\beta_{t-1,j}, g_{j}(r_{t} - E_{t-1,j}(r_{t}))) \]
Forecast Gain Distributions \((g_i)\)

- \(\log_2\) (half life) uniformly distributed
- Range = \([1, 50]\) years
Volatility Forecasts

\[ \varepsilon_{t,j}^2 = (r_t - E_{t-1,j}(r_t))^2 \]

\[ E_{t,j}(\sigma_{t+1}^2) = (1 - g_{j,\sigma})E_{t-1,j}(\sigma_t^2) + g_{j,\sigma}\varepsilon_{t,j}^2 \]
Learning Dynamics: Evolution

- Passive
  - Successful strategies acquire more wealth

- Active
  - Agents shift to “better” forecasting rules
Active Forecast Choice

Agents

Forecasts
Market Clearing

\[ S_{t,i}(P_t) = \text{Share demand} \]

\[ S_t(P_t) = \sum_{i=1}^{l} S_{t,i}(P_t) = 1 \]

\[ S_t(P_t) = \frac{1}{P_t} \sum_{i=1}^{l} (1 - \lambda)\alpha_{t,i}(P_t)W_{t,i}(P_t) \]

\[ \alpha_{t,i} = f(P_t) = f(\text{Info}_t) \]
## Key Parameters (Baseline)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_g$</td>
<td>2%</td>
</tr>
<tr>
<td>$\sigma_d$</td>
<td>12%</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>3.5</td>
</tr>
<tr>
<td>I,J Rules and agents</td>
<td>16000, 4000</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.00004</td>
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<tr>
<td>$g_i$, forecast gain</td>
<td>[1,50] years</td>
</tr>
<tr>
<td>Trim range</td>
<td>[0.075, 0.15]</td>
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<tr>
<td>$m$</td>
<td>0.99</td>
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<tr>
<td>Portfolio Range</td>
<td>[0.05, 0.95]</td>
</tr>
<tr>
<td>P/D Regression</td>
<td>52 weeks</td>
</tr>
</tbody>
</table>
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Simulation Comparisons

- Baseline (Simulation 200,000 weeks)
- Weekly CRSP VW Index (1926-2009)
- Annual S&P + dividends (Shiller, 1871-2009)
Stock Prices

Graph of S&P Level and Simulation Price over years 2000 to 2010.
Weekly Returns

![Graph of Weekly Returns](image)

- VW Returns
- Simulation

Year

VW Returns

Simulation
Return Distributions

VW Weekly: 1926–2009

Simulation Weekly Returns
Weekly Return ACF’s

Return Autocorrelation

Absolute Return Autocorrelation
P/E Ratios (Annual)

Years: 1873–2009

S&P P/E Ratio

Simulation P/E Ratio
## Annual Summary Statistics

(Annual real total returns, S&P 1871-2008, simulation)

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Baseline</th>
<th>P/E</th>
<th>P/D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean(P/D)</td>
<td>22.76</td>
<td>15.32</td>
<td>26.62</td>
</tr>
<tr>
<td>Std.(P/D)</td>
<td>7.56</td>
<td>5.97</td>
<td>13.81</td>
</tr>
<tr>
<td>Autocorrelation</td>
<td>0.74</td>
<td>0.68</td>
<td>0.93</td>
</tr>
<tr>
<td>Return Mean(%)</td>
<td>7.81</td>
<td></td>
<td>7.95</td>
</tr>
<tr>
<td>Log Return Mean(%)</td>
<td>5.82</td>
<td></td>
<td>6.22</td>
</tr>
<tr>
<td>Return Std.</td>
<td>0.19</td>
<td></td>
<td>0.17</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.39</td>
<td></td>
<td>0.30</td>
</tr>
</tbody>
</table>
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Share Demands

\[ z(P_t) = \frac{\sum_{i=1}^{l} \alpha_{t,i}(P_t)(1 - \lambda)W_{t,i}}{P_t} \]

\[ z(P_t) = \frac{(1 - \lambda)\sum_{i=1}^{l} \alpha_{t,i}(P_t)((P_t + D_t)S_{t-1,i} + B_{t-1,i})}{P_t} \]

\[ z(P_t) = (1 - \lambda)\sum_{i=1}^{l} \alpha_{t,i}(P_t)(1 + \frac{D_t}{P_t})S_{t-1,i} + (1 - \lambda)\sum_{i=1}^{l} \frac{\alpha_{t,i}(P_t)B_{t-1,i}}{P_t} \]

\[ z(P_t) \approx (1 - \lambda)\sum_{i=1}^{l} \alpha_{t,i}(P_t)S_{t-1,i} + (1 - \lambda)\sum_{i=1}^{l} \frac{\alpha_{t,i}(P_t)B_{t-1,i}}{P_t} \]

Active \hspace{1cm} \text{Rebalancing}
Crash Demands by Strategy (Shares)

Red: active, Green: rebalance, Blue: total
Importance of Rebalancing

\[ z(P_t) \approx (1 - \lambda) \sum_{i=1}^{I} \alpha_{t,i}(P_t)S_{t-1,i} + (1 - \lambda) \sum_{i=1}^{I} \frac{\alpha_{t,i}(P_t)B_{t-1,i}}{P_t} \]

In general, when volatility (risk) perceptions are low:

- Cash low
- Rebalancing (stabilizing) weak
- Crash is imminent
How Does the Population Contribute to Crashes?

➡️ Wealth across strategies
➡️ Wealth across gain levels (low->high)
➡️ Wealth across volatility gain levels
Gain Wealth Distributions

Adaptive

Fundamental

Noise

Gain: Min <-> Max

Fraction

Gain: Min <-> Max

Fraction

Gain: Min <-> Max

Fraction
Volatility Gain Wealth Distributions

Adaptive

Fraction

Gain: Min <-> Max

Fundamental

Fraction

Gain: Min <-> Max

Noise

Fraction

Gain: Min <-> Max
Why does wealth go to

- Adaptive (trend following) strategies
  - Market timing is difficult for fundamental traders (*Scared*)
- High gain (recent past) strategies
  - Short term risk aversion and volatility persistence
Model Summary/Conclusions

- Minimal platform
- Realistic dynamics
- Instability explanations
  - Volatility perceptions and cash holdings
    - Minksy/Kindleberger
  - Adaptive/high gain forecasts persist
  - Fundamental market timing difficult
Strategy Fractions and Timing

![Graph showing price and fraction over weeks with two lines representing Adaptive and Fundamental strategies.](image-url)
Experiment: Low Gain Only

➡ Forecast gains [40,50] years
Low Gain (Long Memory)

![Graph showing P/D Ratio and Weekly Returns over time.]

- **P/D Ratio**: The graph on the top shows the P/D Ratio over time, with a trend line that fluctuates around a mean value.
- **Weekly Returns**: The middle graph displays the weekly returns over time, showing a fluctuating pattern around zero.
- **Weekly Return Distribution**: The bottom graph represents the distribution of weekly returns, indicating a normal distribution with a slight skew.

The graphs provide insights into the performance metrics over a extended period, illustrating the long-term memory effect in the context of low gain.
Price/Portfolio Comovements

Weeks

Price

Fraction

Adaptive

Fundamental

Noise

DP Rule