Escape Speed of Air Molecules

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We are told that a particle travelling faster than about $11,000 m/s$ has enough kinetic energy to completely escape from the Earth's atmosphere. Although the average speeds of nitrogen, hydrogen, and helium atoms are slower than this, occasionally, some will gain enough kinetic energy to escape. Each of the three atoms mentioned has a different probability of escape, which is determined by the particle's mass. From the escape probabilities, we can do some back of the envelope approximations to determine why these atoms compose specific percentages of the atmosphere.

Let's start with the nitrogen atom. Diatomic nitrogen exists in our atmosphere. Any nitrogen that escapes will come from the upper atmosphere where the temperature is around $1000K$. To find the probability of escape, we need to find the probability of a nitrogen atom reaching a speed greater than $11,000 m/s$. To do this, we will need to evaluate the integral given by the Maxwell distribution from a speed of $11,000 m/s$ to ∞. The integral is

$$P = Probability\ of\ escape(v > 11000 m/s) = \frac{4}{\sqrt{\pi}} \int_{x_{min}}^{\infty} x^2 e^{-x^2} dx. \tag{1}$$

Note that x is given by

$$x = v\sqrt{m/2kT} = v/v_{max}. \tag{2}$$

Let x_{min} be given by $v/\bar{v}$ where $\bar{v}$ is the average speed at $1000K$ and can be found from

$$\bar{v} = \sqrt{\frac{8kT}{\pi m}}. \tag{3}$$

Using these formulas for the diatomic nitrogen molecule, we find

$$\bar{v} = 860 m/s$$

$$P = 8 \times 10^{-70}. \tag{4}$$

$$\tag{5}$$

Similarly, for the helium atom, we find

$$\bar{v} = 2290 m/s$$

$$P = 5 \times 10^{-10}. \tag{6}$$

$$\tag{7}$$

Lastly, for the diatomic hydrogen molecule, we see

$$\begin{aligned} \bar{v} &= 3260m/s \\ P &= 5 \times 10^{-5}. \qquad (8) \\ & \qquad (9) \end{aligned}$$

These results can be interpreted through a rough approximation of the number of molecules in the atmosphere. Based on an atmosphere that extends about $100km$[1], and a uniform density, we can see that there are about 10^{50} molecules in our atmosphere. Now, assuming that a molecule attempts to escape once a day, we can learn how many molecules will escape on average. This shows that nitrogen molecules will rarely ever escape, whereas helium and hydrogen atoms will escape very often. Specifically, the probability of a nitrogen molecule escaping in one day is 10^{-20}. However, up to 10^{40} helium molecules could escape a day and up to 10^{45} hydrogen molecules can escape in a day. These numbers are meant to show how many molecules could escape, not how many actually do. This explains why our atmosphere is mostly nitrogen[2]. However, it does not account for the small percentage of hydrogen gas that currently exists in the atmosphere. Although hydrogen gas escapes regularly, it also diffuses into the atmosphere from other sources, thus replenishing the roughly 1% of the atmosphere that is hydrogen.

A similar procedure could explain why the moon has no atmosphere. Since the escape speed of the moon is only $2400m/s$, we can see that even the nitrogen molecules will easily escape the gravitational pull of the moon with a probability that is even greater than the probability of escape of hydrogen molecules from the Earth. This assumes that the temperature is the same as what we used for the Earth. The probability of escape for the nitrogen molecules on the moon is

$$P = 10^{-3}. \tag{10}$$

Therefore, the moon does not have an atmosphere because any particles that have a chance of becoming part of it's atmosphere can easily escape.

In conclusion, we have used a qualitative model for escape from the atmosphere to explain why the Earth's atmosphere exists as it does, and why the moon's atmosphere is nonexistent.

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[1] http://www.aerospaceweb.org/question/atmosphere/q0090.shtml

[2] http://www.indiana.edu/ geog109/topics/atmosphere/permanent_gases.html