The "Two Cultures", Mathematics and Postmodernism

by Edgar H. Brown, Jr.

September 10, 2004

1. Introduction

Based in part on twentieth century foundations of mathematics we describe a simple way of thinking about knowledge, a sort of minimal epistemology, which illuminates basic differences between the sciences and the humanities and disputes some of the claims of postmodernism. Our analysis is applicable to "bodies of knowledge", which include literary criticism, physics, philosophy, astrology, mathematics, psychology, history, medicine, religion,.... Postmodern thought concerning theories of knowledge, for example, Jean-Francois Lyotard's, *The Postmodern Condition* ([4])*,* Andrew Ross's *Science Wars* ([6]), Richard Rorty's, *Introduction, Objectivity, Relativism, and Truth* ([7]) can be viewed as aggressively disputing C. P. Snow's two culture dichotomy between the humanities and science. Encouraged by Thomas Kuhn's analysis of scientific knowledge in *The Structure of Scientific Revolutions in Science,* Postmoderist doctrines now holds that each body of knowledge has its own internally defined notion of truth or validity and in consequence, the validity of doctrines cannot be compared. Often an analysis of a particular subject matter is deemed to be merely an interpretation based on the realities of power. A good example of the postmodern viewpoint is provided by Andrew Ross, editor of *Science Wars,* in his essay "New Age Technicultures" in *Cultural Studies* ([6]).

> "In this paper [i.e., the distinction between authentic science and pseudoscience] it is worth drawing an analogy between the demarcation lines in science and the borders between hierarchical taste cultures - high, middlebrow, popular - that cultural critics and other experts involved in the business of culture have long had the vocation of supervising. In both cases, we find the same need for experts to police the borders with criteria of inclusion and exclusion. In the wake of Karl Popper's influential work, falsifiability ([5]) is often put forward as a criterion for distinguishing between the truly scientific and pseudoscientific. But such a yardstick is no more objectively adequate and no less mythical a criterion than appeals to, say, aesthetic complexity have been in the history of cultural criticism.

> Falsifiability is a self-referential concept in science in as much as it appeals to those normative codes of science that favor objective authentication of evidence by a supposedly objective observer. In the same way "aesthetic complexity" only makes sense as a criterion of demarcation inasmuch as it refers to assumptions about supposed objectivity of categories like the "aesthetic" refereed by institutionally accredited judges of taste."

Not surprisingly, postmoderist doctrines turn out to be entirely in the humanities as is almost all of philosophy and surprisingly, philosophy of science. More surprisingly, Kuhn's analysis of science, which might be thought to bridge the gap between the humanities and science, is itself largely in the humanities. With considerable justification, postmodernists interpret Kuhn as claiming that the acceptance of a science theory from among competing theories is not decided by experimental evidence, a claim we argue is patently false.

The titles of the remaining sections of this paper are Greek Origins, A Minimal Epistemology, Model-Technology (MT), Predictive Power, MTs for Practical Knowledge and Mathematics, MTs in the Humanities, Science MTs, Kuhn and Postmoderism, Relativity of Knowledge, Meaning, Facts and Laws of Nature, Concluding Comments

2. Greek Origins

We begin with a reading of the history of western philosophy, which finds the two cultures essentially delineated by the Greeks in the 6th through 4th centuries BCE in large part due to their approach to mathematics. A good share of the mathematics one learns in school, arithmetic, solving quadratic and systems of linear equations, calculating areas and volumes, and geometric constructions were known to the Egyptians, Assyrians and Babylonians by the end of the second millennium BCE. This ancient mathematics consisted almost entirely of recipes for performing calculations used in commerce, construction and astronomy, with very little explanation or argument as to why they gave correct or approximately correct answers. Geometry arose in the valley of the Nile to meet the practical need to restore the boundaries of agricultural land, which was obliterated by the annual inundation of the Nile River, giving, in crude form, plane geometry.

The Greek philosopher and scientist, Thales (640 - 546) of Miletus, who had knowledge of Egyptian geometry and Babylon astronomy, is credited with founding mathematics as a deductive science, that is, organizing mathematics around demonstrating by logical arguments the correctness of ones assertions and calculations. In addition he recognized the possibility that material phenomena conformed to discoverable laws, which in the next three centuries were demonstrated to be the case in a variety scientific discoveries. For example, Thales set forth and demonstrated the general angle, side, angle theorem for congruent triangles and used it to measure the distance of ships from shore. Unlike the Egyptians who only had calculation procedures giving answers to practical problems, the Greeks went on to insist on knowing why such procedures worked, based on logical arguments.

Aristotle (384-322), in codifying logical reasoning, set down rules of inference and recognized the importance of axioms for logic, postulates for the subject at hand, definitions of terms and the importance of giving logical arguments starting with the postulates. The treatment of plane geometry in Euclid's "Elements" starts with postulates, a short list of sentences about points and (straight) lines taken to be true. For example, "Through two points there passes exactly one line." Starting with the postulates, repeated applications of rules of inference produce further true sentences, which are the "theorems". The sequence of inferences (the line of reasoning) getting to a theorem is a proof of the theorem.

Logic provides a reasonably straightforward way of verifying that a line of reasoning is correct. Given a collection of assumptions, say the postulates of Euclidean Geometry, it is usually very difficult to determine whether a given statement logically follows from the assumptions. But if someone comes forward with a line of reasoning (a proof) that the statement follows from the assumptions, then it is fairly straight forward to check that the proof is correct; logic provides a step by step way for doing this.

Combining of Aristotle's precise formulation of logic with Thales' method, the main elements of modern science were then in place. The researcher, on the basis of experience of the world (the Egyptian land surveying techniques), develops the concepts of point, line and plane and constructs an abstract theory about them (Euclidean geometry). The engineer utilizes this theory to formulate practical procedures, which he passes on to craftsmen as recipes for tasks. From the vantage point of the theory, additional applications are sought

Indulging in a bit of historical fiction, a funny thing happened on the way to the Agora postponing the rise of experimental science for 2,000 years. While walking to the Agora, Socrates (469-399) thinks of incorporating the deductive reasoning methods of Thales into his pedagogical techniques thus inventing the Socratic dialogue method by which one can derive the true nature of such things as "justice" and "beauty". Plato (428-348), a student of Socrates takes up this technique going so far as to have inscribed over the porch of his Academy: "Let no one who is unacquainted with geometry enter here." It was not because geometry was useful in everyday life that Plato so stressed its study but because geometry trained the mind in correct, abstract thinking. Curiously, this point of view survived well into the 1950's in the teaching of high school Euclidean Geometry without a hint that it had any practical applications, not withstanding that it had, together with its applications to the physical world, no serious competitor as the most important scientific theory and accompanying engineering practices until the seventeenth century and possibly not until the nineteenth. The reason for this neglect was probably that Plato's Academy was training the ruling elite who would focus on morality, religion and politics.

An essential element of science is direct observation and interaction with the world. Plato set forth a very different doctrine, to the effect that knowledge cannot be derived from the senses; real knowledge only has to do with concepts. The senses only deceive us; hence we should, in acquiring knowledge, ignore sense impressions and develop reason. This raises the problem of "universals". If our knowledge is to consist of true assertions, then words such as "man", "God", "soul", "straight line", "color" and "justice" should be the names of things. Plato addresses this problem by postulating two worlds, out there! The world of the senses, the material world, and the world of ideas, of ideal objects. Euclidean geometry lends itself very well to this point of view. It supplied wonderfully solid truth about the world and who could doubt the truth of theorems in geometry. With respect to its subject matter, we draw a straight line on the blackboard, but it is only a crude approximation to a real, ideal straight line out there. Thus the existence of the ideal objects of geometry lends support to postulating the existence, out in the world, of other ideal objects such as justice, moral laws and gods.

As Aristotle emphasized with his idea of postulates, to get truth about the world using logical arguments, one needs truth inputs to get truth outputs. In Plato's system where does the truth inputs come from? Again geometry comes to the rescue. Just thinking about it, and possibly engaging in Socratic dialog, one comes to know about those straight lines and points out there and that through two points there passes exactly one straight line, quite independently of our stretching strings from poles to mark out the boundary of our garden. The emphasis is on deep thinking, convincing arguments, Socratic dialogue, rhetoric and sometimes sacred texts or wisdom.

Adding Aristotle's logic to Plato's ideal worlds gives the prototype of the underlying epistemology of the humanities to the present day, with some exception, e.g. Derrida ([1]) who calls it logocentrism. The subsequent course of Plato’s approach, which we call the *Platonic Form* of knowledge, occupies the core of a sequence of not very drastic variations of Plato’s “truth imputes”, his two worlds and, in Kant’s sense, a priori knowledge about the ideal world. Some of the major variations in this line of thought include the works of Plotinus (204-270 C.E.), Descartes (1596-1650), Spinoza (1624-1677), Leibniz (1646-1716), Berkeley (1685-1753), Kant (1724-1804), Hegel (1770-1831), Marx (1818-1883), Heidegger (1889-1976) and Husserl (1859-1938). In some cases, postulates are quite explicit, for example, for Descartes “I think therefore I am,” is a postulate having as a consequence “Something exists,” which he levers up to “God exists,” using “those long chains composed of very simple and easy reasoning, which geometers customarily use to arrive at their most difficult demonstrations.” (Descartes’ *Discourse on Method*.) More often postulates are not explicit, being replaced by manipulating concepts. For example, the ontological proof of the existence of God goes as follows:

The essence of anything consists in the collection of qualities it possesses. A thing or being is inferior to another being having the same qualities plus the quality of existing. Hence if God did not exist, there would be an essence superior to him having his qualities plus existing. Therefore being the most perfect being, he must have the quality of existing. Hence God exists. Wow!

Over the last 2,300 years, Plato's "ideal world" goes through of a variety of transformations, various aspects of consciousness being a popular contemporary substitute. For example, after 2,300 years, Husserl is marvelously Platonic in his expectation that transdental-phenomenological -consciousness will be able to "intuit the essences of objects and to thereby establish the universal structures of human consciousness providing the grounds for procuring absolutely certain and valid knowledge about things and events." (J. J. Kockelmans. A First Introduction to Husserl's Phenomenology).

We list some feature of the Platonic Form, which play an important role in the humanities. Both the humanities and science utilize logic, preferring to avoid logical contradictions. (Or at least we restrict our attention to those that do.)

(1) Knowledge consists of true statements about the world, where world is taken in the broadest sense.

(2) Until very recently, that God exists should be included among the potentially true statements and preferably turn out to be provably true.

(3) More recently, various religious-political doctrines, Kantian philosophy, Hegelian dialectical idealism, dialectical materialism, free-enterprise, claim to be true by their adherents, on Platonic Form type grounds, that is, by virtue of deep thinking and logical argument.

(5) In their more pristine forms, a doctrine consists of a collection of clearly true statements, the postulates, and logical deductions from them. Great weight is given to propositions that can be proved to be true.

(6) Language is fully used with its concepts about the world, their interrelations and their connections to the material world. "This is an apple," coupled with holding up certain objects, plays a role. But the doctrines are not crucially confounded by observations and interaction with the world. Acquaintance with the world and deep thinking provides the basis for truth, which is elaborated through logical arguments.

We will argue that none of the features, (1) through (6), of the humanities culture, play a significant role in the experimental sciences.

3. A Minimal Epistemology

We will view knowledge as broken up into fairly self-contained units dealing with large or possibly small aspects of the world including human activities in the broadest possible sense. Roughly speaking our minimal epistemology will view a body of knowledge as made up of two parts, a language in which the elementary rules of logical reasoning apply, for example, one may deduce from “All men are mortal and Socrates is a man” that “Socrates is mortal.” Second, that the language is related to the world, an idea to be made precise, but includes the notion that someone holding up an object for inspection and declaring, “This is apple,” may have some relevance to knowledge about apples. Although the practitioners of some body of knowledge may view singling out these aspects as being of little interest, the claim is that in almost all cases they are nevertheless present.

A major difficulty in setting down our two aspects of knowledge is the immense baggage these ideas carry. Even simple things such as elementary logic have been discussed and explained from a variety of perspectives, including some exceedingly murky formulations, i.e. in Ludwig Wittgenstein’s, *Tractatus Logico-Philosophicus* ([9]). In the past a great deal of agonizing went on about what the subject of logic should be, in particular, about the idea of a “proposition”.

One wishes to talk about statements such as: “Silver is a metal,” or “My necktie is red,” that is, states of the world. But one cannot merely settle for states of the world because one must give attention to “My necktie is red,” even if my necktie is blue, that is, when “My necktie is red” is false. One of many approaches to resolving this difficulty is to define a proposition as an abstract entity about which one may have a belief. Fortunately, all of this has been swept away, as far as logic is concerned, with the invention of formal languages, an amazingly simple, and as it turned out, powerful idea coming out of the foundations of mathematics. The idea is to concentrate on sentences, those things made out of sounds or ink marks on paper.

Formal languages, for example, as in Russell and Whitehead's *Principia Mathematica*, enable us to set forth the rules of logical reasoning while entirely disengaging ourselves from tangles about "meaning" and from metaphysics, most particularly ontology, the study of being or the essence of things.

To carry out our arguments we need a precise formulation of our two parts of that is separated as much as possible from last 2,500 years of western philosophy. Although superficially similar to logical positivism, our two parts formulation only shares the idea of a formal language. The exposition of our minimal epistemology is meant to offer a way of looking at knowledge, which is reasonably unambiguous and applicable to both the humanities and the sciences.

For our purposes, a formal language is a finite collection of symbols including symbols for predicates, nouns and pronouns as well as the logic symbols, "and", "or", "For all" together with rules for forming sentences from sequences of symbols and rules for carrying out logical reasoning within the formal language. The point is to get away from "propositions" by having "sentences" as meaningless physical objects such as scrabble pieces lined up in a row.

In our minimal epistemology, a unit of knowledge consists of a formal language together with a non-linguistic part consisting of a collection of procedures for interacting with the world, that is, making measurements in the broadest sense. The idea of a procedure includes the existence of a collection of experts with a teachable expertise who can with some success, identify situations where the unit of knowledge is applicable and in these situations execute its procedures, producing as their outcomes, sentences in the model. We view the execution of a procedure as a well defined interaction between the expert and the world whose outcome is a sentence such as "That chair is blue", "The temperature of this water is 126 F°", "Your astrological reading for today is, 'Avoid financial decisions'", "There are three apples in the bowl", "Rembrandt was a greater artist than Pollock", "The distance between the earth and the moon is 240,000 miles", "My wife said I should mow the lawn." Procedures can be quite technical or superficially, quite mundane as in the case of a procedure whose execution yields "That chair is blue." We use "Procedure" as a general name for the procedures associated with a body of knowledge.

Actually our minimal epistemology is more accurately described as associating to a body of knowledge, in the usual sense, a formal rendering of the body of knowledge, consisting of a two part formulation as described above, capturing, we claim, most of its meaning and content. We call the linguistic model of a unit of knowledge its *model*, its collection of Procedures its *technology* and we call the formal unit of knowledge so specified an *MT*. For example the “Land-Surveying” MT has a formalization of Euclidean geometry as its model and its technologies, the collection of procedures surveyors use, i.e. measuring distance with a long tape measure and vertical and horizontal angles with a transit. The MT that produces “That chair is green,” might be formulated in a way that includes that amazingly complex measuring device which takes months to learn how to operate, namely, our vision apparatus. In subsequent sections we will spell out in more detail our notions of a model and a technology and how they should be interrelated so as to form an MT. Roughly speaking, a Procedure and its possible outcomes when it is executed, are sentences in the model. For example, the Procedure named “The temperature of x is y,” when executed might produce “The temperature of this steel bar is 700° F,” where these two sentences are in the model.

Qualifying as a MT is not a very stringent requirement on anything passing for knowledge, especially if we allow the model or the technology to be the empty set so as to include pure mathematics and riding a bicycle. The Procedure “x is an apple” has possible outcomes “This is an apple” and “This is not an apple,” and it makes an important semantic contribution to the MT going to the meaning of “apple”. The MT having as an axiom or a theorem “Apples exist” does say something about the world. In contrast, the theorem that the sum of the angles of a triangle is 180° in the Land-Surveying MT yields a more important example. Suppose “A”, “B” and “C” denote markers on three hill tops and angles are denoted by “ABC” for the angle given by the lines AB and BC. Suppose executions of the angle measuring procedure yield “ABC=26°” and “BCA=70°”. The MT then says that of the possible values yielded by measuring BAC, all values between 0°

and 180°, the one that will be produced is 180° - 26° - 70° = 84. This example exhibits the power of prediction of the Land-Surveying MT, something considerably stronger than simple existence claims. Although we will very much elaborate this example of predictive power, it actually tells the whole story as to the difference between the two cultures and goes a long way toward accounting for the vast expansion of science and technology.

Essentially, predictive power replaces the properties 1 through 6 previously ascribed to the humanities culture. It is the basis for the human project of trying to control the natural world for human purposes while the humanities might be described as devoted to shaping human institutions.

In keeping with our classical roots, most people including scientists are vaguely Platonic in their outlook. The scientists view themselves as discovering laws or approximate laws of nature, which are true statements about what is out there in the world. But the power of accurate predictions is paramount. Fussing about with what it means to say that Maxwell's equations are true is of little concern. The scientists view is that there should be no difficulty in developing an adequate theory of truth that accommodates the primacy of experimental evidence. Tarski's semantic definition of "true", "Today is Wednesday," is true if and only if today is Wednesday ([8]), would seem to be quite adequate. This contrasts with the Platonic Form where the nature of truth is a major concern holding out the possibility of "truths of complete certainty" and devoting immense energy to setting forth the meaning of truth.

4. Model-Technology, MTs

We give a fairly complete and accurate description of our MT concept by presenting a simple example. The ingredients of the model, a formal language, is a finite set of symbols including "not", "and" and "for all", for logic, and symbols for proper nouns, pronouns and predicates for their intended subject matter but without attaching any explicit meaning to the symbols. One has in mind and is guided by meanings for the symbols but the combination and manipulation rules for them should not involve meaning. To emphasis this we use bold for " ", i.e. **and** for "and".

We name our MT "Mortality". The symbols are **not**, **and**, **for all**, **is a man, is mortal**, **Socrates** and **x**. We would use **it** instead of **x** but for the gender distinction**.** Often one must refer to more than one **it** in which case one uses **x**, **y**, **z**.... The grammar for Mortality goes as follows: The *constants* (proper nouns), in this example, only one, **Socrates** and the *variables* (pronouns), again only one, **x.** Together they comprise *objects*. The *predicates* are **is a man** and **is mortal**. If a is an object and A is a predicate, aA is a formula (thus, for example, **x is mortal** and **Socrates is a man** are formulas.). If p and q are formulas, **(not** p**)** and **(**p **and** q**)** are formulas. (Add **(** and **)** to the symbols.) If p is a formula and v is a variable and **for all (**v**)** is not in p, then **for all(**v**)**p is a formula. If **for all(**v**)** is in a formula p, then v is said to be a bound variable of p, otherwise it is said to be a free variable of p. A formula is a sentence if it contains no free variables. The usual terminology for this sort of construction is that Mortality is the object language and English, including a, v, A, p and q, is the metalanguage used to discuss Mortality. Additional logic symbols are introduce as abbreviations, i.e., p **or** q for **not(not** p **and not** q**),** p **implies** q for **(not** p**) or** q, and **for some**, for **not for all(**v)**not**. One then chooses a collection of axioms (for logic) which when expanded using the rules of inference give all the "logically true sentences". The axioms might include, for all sentences p, p **or not** p and for all variables v, constants a and predicates A, **for all(**v)vA **implies** aA. (i.e. **for all (x)(x is mortal) implies (Socrates is mortal)**). The postulates are: **for all(x)(x is a man implies x is mortal).** The construction **for all(x)(x is a man)** avoids the muddle Aristotle introduced by making all men the same kind of object as Socrates; **for all(x)(x is a man implies x is mortal)** replaces "All men are mortal". Rules of inference are: Axioms and postulates are theorems and if p and p **implies** q are theorems, then q is a theorem. A proof of theorem p is a step by step sequence of inferences stating from an axiom or postulate and ending at p. Mortality has predictive power, as follows: The **is a man** expert encounters Socrates, exercises his expertise and produces **Socrates is a man** and predicts from the postulate that Socrates is mortal. Alternatively, he adds **Socrates is a man** to Mortality, thus declaring that **Socrates is a man** is true and then notes that **Socrates is mortal** is a theorem, a proof being: **for all(x) (x is a man) implies (x is mortal)** and **(for all(x) (x is a man) implies (x is mortal)) implies ((Socrates is a man)**

implies (Socrates is mortal)) are axioms. Hence **(Socrates is a man) implies (Socrates is mortal)** is a theorem. **Socrates is a man** is an axiom and hence **Socrates is mortal** is a theorem. He then predicts that executing **is mortal** on Socrates would produce, from the possible outcomes **Socrates is mortal** and **not Socrates is mortal**, **Socrates is mortal**.

We emphasis that the symbols in Mortality, on its own, are not assigned any meaning. Meaning comes from the Procedures. The expert, pointing while repeatedly executing **x is a man** produces **That is a man, That is a man, That is not a man, That is a man**... which contributes to the meaning of **man** (We need another constant, **That**.) But if someone comes forward with a sequences of symbols, one can check mechanically whether the sequence is a sentence, and if someone comes forward with a sequence of sentences which he asserts is a proof (a logically correct line of reasoning) one can mechanically check whether his reasoning is correct.

Although our notion of an MT is fairly precise, typically an MT is very substantially ambiguous, mostly due to uncertainty as to the range of applicability of its Procedures. In Mortality, the procedure **is mortal** might be questioned. To clarify the meaning of "mortal" one would construct another MT again with its own ambiguities. In other words, in critically examining some aspect of a model, one constructs another model. It is like the question, what holds up the earth. It sets on the back of a large turtle. And what holds up the large turtle. It sets on another large turtle and it is large turtles all the way down. Similarly, it's models all the way down.

5. Predictive Power

Our MT formulation makes possible a quite precise definition of predictive power, which will be useful in differentiating the two cultures. To explain this idea we use a very simple science MT example, named "Water-Boiling", simpler than Land-Surveying. Consider the well-known bit of knowledge that water boils at 212º F or, more accurately, water, in its liquid state, is boiling, if and only if its temperature is 212º Fahrenheit. Water-Boiling has predicates, **has temperature** and **is boiling**, a constant **S,** variables **x**, **y** and **z** (We think of **S** as the name of some water at hand and the variables, as pronouns for quantities of water in various states and temperatures) and one postulate**,**

(For all x) ((x has temperature 212ºF) if and only if (x is boiling)))

The technology consists in the Procedures **x has temperature y** and **x is boiling.** Then, for example,

(S has temperature 126ºF) implies not(S is boiling)

is a theorem.

If the user measures the temperature of his flask of water and produces **(S has temperature 126ºF)**, adds **S has temperature 126ºF** to the model as a postulate (declaring it to be true), the prediction **not(S is boiling),** a theorem, is obtained. He might verify this prediction if a further measurement (he looks at the water), yields **not(S is boiling)** (no steam and bubbles).

We extend the notion of Procedures by including formulas formed by combining Procedures with **and**, **or**, **implies** and **not**; for example,

((x is boiling)and(x has temperature of yºF))

To execute this Procedure one simultaneously measures whether x is boiling and its temperature.

By virtue of the logic axioms, an MTs language can itself be very rich in theorems containing as it would logically true statements involving the non-logic words. The language thus provides plenty of opportunities for constructions both technical and social, introducing concepts and their interrelations, but theorems produced without using its postulates could not be contradicted by the outcome of executing any of its Procedures. There is the possibility that the execution of a Procedure will yield a sentence contradicting the model, that is, a sentence p such that **not** p is a theorem whose proof uses the postulates. For example, in the Boiling-Water MT, the Procedure

(x has temperature of yºF)implies(x is boiling)

has the property that when it is executed either it produces a theorem, say

(The temperature of S = 126ºF)implies not(S is boiling)

(S is found to be at temperature 126ºF and not boiling.) or a sentence contradicting the M of the MT, say

(S has temperature 186ºF) implies (S is boiling)

We define a Procedure of an MT to be a *predictive Procedure* if it has the property that some of its possible outcomes are theorems and some contradict the MT's model, an expert can judge with considerable accuracy situation in which it can be successfully used, and in the past, after considerable use, it has consistently produced theorems. An MT has *predictive power* if it has predictive Procedures (Having some of its Procedures regularly producing contradictions is not excluded.)

One could have an MT with most of its theorems unrelated to its predictive Procedures. We define an MT to be *mature* if each of its non-logic axioms are contradicted by the negation of one of the outcomes of one of its predictive Procedures and vigorous efforts have failed to produce Procedure outcomes contradicting its model. We call matured MTs the *Thales Form* of knowledge.

The Thales Form of knowledge is closely related to Popper's definition of empirically valid science. An enduring problem for the epistemology of science has been the problematic nature of inductive reasoning. From the sun's having risen every day can we conclude the "law", "The sun rises every day." One would seem to need this if one wants the laws of science to be true statements about the world. Popper proposed that the substitute for induction should be failure to falsify, that is, in our terms, failure to find Procedure outcomes contradicting the model.

Of course the power of experimental science is its ability to make accurate predictions. On the basis of Procedures applied to a specific situation, a prediction is made as to the outcome of executing other Procedures and if these Procedures are executed, the predictions prove correct. If the temperature of the water is 212ºF, it is boiling. If a prediction failed to be accurate it would provide a falsification of the MT in Popper's sense. Thus our mature MT is the same as Popper's empirically valid with a positive spin.

Of course some "sciences" are about the world in the sense of using names of things in the world but fail to have predictive Procedures. Almost anything that passes for knowledge can, with some accuracy, be described in terms of our MTs. For example, astrology has an elaborate formal model involving the positions of the heavenly bodies and human events along with Procedures yielding such sentences as "Mary Smith was born at 6:37 AM, July 19, 1964," "Susan Jones will receive some good news this week," and "September fifteenth is an auspicious day for Harry Margulis to begin production on his next film," but astrology is a good example of a unit of knowledge with no predictive Procedures. In astrology, a prediction gives a tendency for a broadly described event to occur with the consequence that no possible outcome contradicts the model. Given a person's birth date and the present date, astrology might predict that the next week will be a good time to form a new friendship. No turn of events could contradict this in as much as there would always be plausible accounts for any outcome.

Many MTs of an applied sort have predictions based on plausibility rather predictive Procedures. Like astrology, they have Procedures all of whose outcomes are theorems. This occurs a great deal in medicine. On the basis of patient reported symptoms and medical tests, the model provides a diagnosis, a treatment and predicted outcomes, which include the possibility that diagnosis was incorrect or the treatment failed in this instance. The recent emphasis on statistical analysis of treatment outcomes is making more of medicine's MTs have predictive power.

6. MTs for Practical Knowledge and Mathematics

All of our ordinary knowledge about the material and natural world of a more or less technical sort, how to make beer, build houses, grow crops… have straightforward mature MT formulations. More or less, put a handbook in bold type and set down the Procedures, which are described or implicit.

Pure mathematics has a model (see below) but no Procedures. Rather, it has sort of pseudo-procedures, Procedures involving interactions with other models. For example Number Theory has as its subject matter the whole numbers, which in turn are given by a model. The extent to which a subject is understood and worked out depends of course on success in proving fundamental theorems.

Developing conceptual frameworks is one of mathematics' principal functions, but typically the results are put to use in quite different ways than are Platonic Forms of knowledge and most modern and postmodern ideological doctrines. Most of mathematics has applications to other mathematics and to science and engineering. For instance, fluid mechanics is a branch of applied mathematics whose results include a design for the cross section of an airplane wing. This design has been used as a starting point in designing actual airplane wings. Such designs are of course tested in wind tunnels and actual airplanes and appropriately modified. The fact that the design is the conclusion of a theorem in fluid mechanics theory only suggests that it is worth looking at. Being a theorem gives a statement very little authority, except in mathematics or in a well-established MT. In contrast with most ideologies, for example, Free Enterprise doctrines or Marxism, no one passionately insists that airplane wings must use the fluid mechanics design because it can in some sense be proved.

Any mathematician can give several models for the whole numbers, which have as theorems everything we know about numbers including "2 + 2 = 4". But what about counting things in the real world? Aren't numbers names of some things in the world, isn't 2 + 2 = 4 a statement about counting in the world? Apple harvesters, George and Bill, come in with baskets of apples which they empty into a central collection basket. The foreman uses the simple model consisting of the addition tables he learned in school. He adds to his model **B**, **G** and **T** for the number of Bill's, George's and total apples and the axiom **B + G = T**. He asks Bill and George how many apples they brought in and they each respond **2**. He has the theorem **(A = 2 and B = 2) implies (T=4)**. And hence his model predicts **T = 4**. Just to be sure, the foreman counts the number of apples in the collection basket. Suppose he finds that there are five. This may reflect a serious flaw in our theory of counting or a major shift in the structure of the universe. Possibly, but a more likely explanation is someone made a mistake. There are many situations where our counting theory works well and many where it doesn't, measuring a volume of cocoa by counting rounded teaspoons of cocoa. When there is a failure we blame the situation not the theorem.

In modern logic, one has the option of eliminating predicates by replacing "Socrates is a man" by "Socrates is a member of the set consisting of all men". If **man** is read as the set of all men**, x is a man** may be replaced by **x ∈ man.** We now add to the definition of a model the requirement that it contain **not**, **and**, **for all**, **x**, **y**, **z**,..., = and ∈ and **;** a ∈ b is read a is a member of the set b. Also require the Model to include axioms appropriate for logic and ∈ (set theory), for example the

Bernay-Gödel formulation ([2])). The Model part of an MT will then, in addition, have a collection of constants, informally related to the subject matter being described, and postulates involving them. We continue to use predicates.

It turns out that my virtue of the addition of = and ∈, the model will contain all of pure mathematics, in particular it has numbers. The development of this conception of a formal language and the sorting out of the details to produce Bernays-Gödel took many years to accomplish and it is one of the major intellectual achievements of the twentieth century. It played an important role in the development of the foundations of mathematics, of modern analytic philosophy, linguistics and computer science. The development of such a formal language for set theory has a long history involving a variety of difficult technical issues, some of the major players being Frege, Hilbert, Russell and Whitehead and Gödel. It provides a precise formulation of the rules of logical reasoning used by mathematicians in a language that is rich enough and flexible enough for them to say whatever they want to say, at least for the present.

7. MTs in the Humanities

We have described science as being concerned with the material and natural worlds. We take the humanities to be mainly concerned with the *human world*, which we define to be the minds-brains of all humans living or dead and all human works including documents, a world that of course changes over time. The MTs associated with bits of knowledge of the human world are rich in Procedures which reflect basic features of humans as part of the natural world or they may be descriptive of human social constructions, **Person x is in mood y**, **selling marijuana in country x is legal**. By and large, Procedures of this sort have their natural home in the social sciences, function as described in the section on MTs in ordinary knowledge and aim to be in the Thales Form of knowledge. Although knowledge of the human world in this sense plays a very large role in the humanities, providing subject matter and inspiration, knowledge in the humanities is almost always in the Platonic Form with concepts and their interrelations having their origin in the human world but very rarely are there assertions which can be contradicted by experimental evidence, that is, by the execution of a Procedure.

Recall, a basic feature of the Platonic Form of knowledge takes experience of the world - material, natural and human – as inspirational, but claims that true knowledge is to be achieved only by some form of rational thought. Our notion of a model captures the logical reasoning part of rational thought but as Aristotle explained, to get anywhere with this approach one needs postulates, in some way or another. Typically big system philosophers such as Descartes explicitly state postulates and call them First Principals. Kant's moral philosophy offers a clear example of the First Principals of a priori knowledge (see below.) By far the more common approach is to have in hand a large, vaguely defined and hopefully consistent collection of statements that are taken to be true and which play the role of postulates. One then makes logical deductions from them. Branches of the law provide clear examples of this approach. Over the passage of time modifications resolve contradictions and accommodate new, unforeseen situations. Most philosophy papers are also of this form, that is, a subject area has a history including interrelated terminology with somewhat floating, undefined words and a collection of problems. Papers review the previous work and set forth new ways of looking at the subject and set forth new formulations of the problem and/or solutions.

The various philosophical doctrines in the Plato to Husseral line we spoke of can be easily transformed into MT form. Spinoza actually writes in this from. The main trend in the evolution of this line was to transfer Plato's Ideal world, out there, into our heads. For example, Descarte's mind body dualism. This transformation is fully realized in German Idealism initiated by Kant and continued with small variations through Husseral and Continental Philosophy.

As an example we discuss Kant's *Metaphyisic of Morals*.

From an MT point of view, where does Kant get his postulates? Answer, deep thinking. Some things we know from experience, as in the sciences, some things by virtue logic, as in mathematics, and some things we know by our *rational will*, providing *a priori* truths. When we discover such a truth, our reason tells us it is true and requires us to act in conformity with its being true. Its is an *imperative*. It may be a *conditional* *imperative* meaning some conditions must be met before it is compelling or it may be unconditional, a *categorical* imperative. Kant views the natural world and morality as governed by laws. With regard to morality there is one basic categorical imperative: "Act only by a maxim by which you can at the same time, will that it should become a general law." We abstain from commenting on the merits of the categorical imperative and instead contrast how a humanities person and a science person would deal with it. In the humanities a rich discussion involving manipulation of concepts and logical reasoning on Kant's ideas continue to this day. The scientists response to this, as well as to the ontological proof on the existence of god, would be, ok, I see the argument but so what? Give me a concrete example were this would predict the outcome of an interaction of me, or someone, and the world.

A possibly unintended "theorem" in Kant's theory is that reason dictates that one should neither pay nor receive interest on barrowed money. The scientist would respond with, "I'll fix that by modifying the model a bit so it gave the "right" answer." But the categorical imperative analysis was supposed to provide *categorical truth.*

A possible way of looking at a work in the humanities is as having evolving MTs where there is a give and take between the M and T where the M provides a framework for adjudicating or informing culture and human relations and M is modified in response to new or unanticipated human interaction situations.

A contrasting point of view is offered in *Working with Structuralism,* by David Lodge (a participant in this subject of exceptional clarity), giving a marvelous Postmodernist rendering of the Platonic Form:

> "Saussure defined the verbal sign, or word, as the union of the signifier (that is, a sound or written symbolization of the word) and a signified (that is, a concept) and asserted that the relation between the signifier and the signified is an arbitrary one. That is, there is no natural or necessary reason why the sound *cat* should denote a feline quadruped …. . This nucleus of arbitrariness at the heart of language means that it is the systematic relation between words that enable them to communicate rather than the relation between words and things; and it exposes

> the idea of any resemblance between words and things as an illusion. Since language provides a model for all systems of signs, the idea has profound implications for the study of culture as a whole. In brief, it implies the priority of form over content, of signifier over signified."

One may hope that somewhere in the structuralism literature there is a detailed account of what is meant by "concept" and that the last sentence of the above quotation does not mean that the signified is completely abandoned.

8. Science MTs

There are a tremendous number of mature MTs similar to our Land-Surveying example in sciences and engineering. Hundreds of such models go into the design and manufacture of an automobile and produce a vehicle, which will, in most cases, operate as predicted for more than ten thousand miles. Groups of small models like our example are organized into very large ones. For example, a freshman physics text typically has about fifty sections divided among mechanics, heat, sound, light, and electricity and magnetism, each section giving a small model and all of them linked together by the overriding concepts of distance, mass, and time.

Well-established MTs such as our Land-Surveying may be incorporated into ever increasing syntheses, but they are very rarely superseded. Whatever happens to the Theory of Everything, people will continue to use it. The inclusion of the above topics in freshman physics texts was unchanged by the advent of relativity theory and quantum mechanics and will continue to provide important MTs. The progress of experimental science has been a process of refinement with very little lost along the way. The advent of Relativity Theory put limits on the applicability of the Newtonian mechanics model and reconciled Newtonian mechanics and Maxwell's equations for electromagnetism. Quantum mechanics, developed within this framework, greatly expanded this body of knowledge by providing an amazingly successful model for many previously inexplicable phenomena. For example, salt held in a flame produces bright yellow light instead of white light as was predicted by the model based on Newtonian mechanics and Maxwell's equations, but is accounted for by quantum mechanics.

Our MT formulation gives some perspective on the notion that apples are real objects, in contrast to electrons, which are constructs of physicists. If one asked an experimental physicist about electrons, he might respond by rummaging in his desk to bring forth a photograph with a black background and a lot of white lines and curves emanating from a single point, indicate one of the lines and say "There's the path of an electron,” To the question, "Does that electron have the same reality as that apple on your desk", he might respond as follows: The apple and the electron share the theoretical machinery that goes into the concept of "thingness" and the electron needs a more elaborate theoretical framework (model). But the detector used to see the electron is childishly simple compared to the detector needed for an apple, namely, our brains and eyes. Someone might respond with, "But it looks like an apple!" We make him stand in the corner. To put these two concepts, apple and electron, on a more equal footing think of two robots designed to detect apples and electrons, respectively. A robot has a computer as brain, hard drive being its memory and the processor and software its thinking and operating machinery and various sensing devices and hands, arms and legs by which it interacts with its surroundings. Is there any basic difference between the two robots? The picture of the robot is quite telling; there it is, interacting with the world, acting on the world and receiving inputs to its detectors from the world and forming models on its hard drive. Better still; picture a group of robots interacting with the world including each other. Such a “society” could have knowledge in the sense here described.

9. Kuhn and Postmodernism

In Thomas Kuhn’s *The Structure of Scientific Revolutions in Science* ([3]), scientific research that extends the applicability of a generally accepted and perfected theory to previously unexplored situations is called “normal science”, “puzzle solving”. Replacing a previously accepted theory by a significantly different theory is a “revolution” --Ptolemy to Copernicus, Newton to Einstein, Newton-Maxwell to quantum mechanics. Although he offers various factors that might influence choosing one theory over another, he seems to say that in revolutions, the choice of which theory is accepted as valid from competing theories is not decided by experimental evidence, a claim much utilized by postmodernist to justify the notion that each body of knowledge has its own notion of truth applicable only to itself and that notions such as predictive power is simply not applicable outside of science. What Kuhn actually says is more complicated than this but not easily sorted out. In his 1969 "clarifications", he writes, "There is no natural algorithm for theory-choice, no systematic decision procedure which, properly applied, must lead each individual in the group to the same decision..... What one must understand, however, is the manner in which a particular set of shared values interact with the particular experiences shared by a community of specialists to ensure that most members of the group will ultimately find one set of arguments rather than another decisive." This is a sociology and /or history of science claim that I feel no need to address in as much as there is a straightforward historically accurate algorithm all scientists agree on, which we next describe.

We offer a simple criteria: MT A is better than MT B if A's predictive Procedures include all of those of B's and A has some not shared by B. During the making of a choice between A and B a certain amount of turmoil goes on, but in the end the better of the two is chosen. This is certainly the case in the revolutions sighted by Kuhn.

If A and B share predictive Procedures and each has some not shared, keep them both. We do not recklessly through away power, and in the fullness of time A and B may be subsumed under C. Although all of the elements of our MT formulation are contained in Kuhn's work, our main criticism of Kuhn is he is analyzing science from the vantage point of the humanities culture; he implicitly focuses on how the "truth" of a theory is determined where in our terminology "theory" is "model".

In *La Condition Postmoderne*, Jean-Fransois Lyotard very succinctly captures the relation between science and a significant branch of postmodernism with his statement, "To the extent that science does not restrict itself to stating useful regularities and seeks the truth, it is obliged to legitimate the rules of its own game," a marvelously Platonic assertion. It is a perfectly accurate statement if one views all bodies of knowledge as being in the Platonic Form, as does "Continental Philosophy". Our claim is that each body of knowledge has an associated MT, which may or may not have predictive power, in the precise sense we specified, dependent or independent of cultural conventions, and that the extent of predictive power is an important feature. Discovering and stating those regularities is what science is all about. Those regularities manifest themselves in interrelations between the outcomes of Procedures. Aggressively pursuing those regularities beginning about six centuries ago has transformed our world.

In contrast to Lyotard, our MT formulation can be viewed as formalizing Rorty's Pragmatism which is mostly the default setting after rejecting the representation theory of knowledge (empiricism) – "That chair is blue" is true because "That chair" and "blue" correspond to somethings in the world and is true by virtue of their being related by "is".

10. Relativity of Knowledge

Dependence on point of view is completely pervasive in our interactions with the world. Our interactions always depend on our physically limited ways of interacting with the world, our various ways of overcoming these limits and our personal, social and cultural vantage points from which we make sense of and give meaning to these interactions. The question is how, and how successfully, do we correct for this dependence. The Platonic and Thales Forms of knowledge, although in different ways, are completely dependent on these interactions.

A very simple, everyday example of how we deal with differing viewpoints is provided by the weight of physical objects, the points of view being the units of measurement, for example, pounds versus grams. We are perfectly used to the idea that a physical object has weight even though we need a unit in order to measure it and to say what its weight is. When one says something weighs ten pounds we are announcing our point of view, pounds, and its weight from this point of view. In an isolated culture using pounds, this could go unnoticed, but confronted with another culture using grams, some might declare that weight is subjective nonsense or a cultural artifact. A more subtle approach to this situation is to say that the weight of an object is a number and a unit of measurement, together with a standard procedure for converting measurements when the unit of measurement is change (Our culture has an institution, the Bureau of Weights and Measures, which kept a standard one pound object and sold copies of it.) A more sophisticated treatment of weight would say that the weight of an object is an equivalence class of pairs consisting of a number and a physical object providing the unit, where equivalence is defined in terms of the above conversion procedure producing a definition of weight that is independent of the choice of unit. Or saying the same thing in simpler terms, one expresses the weight of objects with respect to a fixed object taken as a unit, recognizing that the choice of unit is arbitrary.

Different languages also provide examples of dependence on point of view and translations provide the method of correcting for this dependence. We use this idea of translating one language into another to compare different units of knowledge. To simplify the discussion we continue to assume all our models contain the same formal rendering of logic and Bernays-Gödel - set theory.

Recall an MT has a model consisting of a formal language having symbols for logic and for nouns and predicates for the subject at hand, axioms for logic and postulates for the subject. In addition an MT has a technology, a collection of non-linguistic elements, Procedures, ways humans interact with the world that are named by formulas in the model and when executed produce sentences in the model, i.e. **x is a man**, when confronted with Socrates, produces **Socrates is a man**.

Define the language of an MT U to be the MT, denoted by LU, where LU's model is U's model with its postulates omitted and LU's Procedures, are the Procedures of U. LU has no non-logic theorems but has Procedures giving meaning to some of its nouns and predicates. Omitting technicalities, we informally define a translation of an MT U_1 into an MT U_2 to be a function which assigns to each formula in LU_1 a formula in LU_2, preserving the logic, set theory and grammar, taking the names of Procedures of U_1 to names of Procedures of U_2 such that experimentation has reasonably established that when a Procedure p of U_1 is executed it produces a sentence whose translation is produced by the translation of p. Define (more or less) a translation between U_1 and U_2 to be a translation from U_1 to U_2 such that each formula of U_2 is the translate of a unique U_1 formula and it is a U_2 Procedure if it is a translation of a U_1 Procedure.

We turn to comparing viewpoints as embodied in two MTs. Define two MTs, U_1 and U_2, to be strictly equivalent if there is a translation between their underlying languages taking axioms into theorems in both directions (This implies theorems go to theorems in both directions.) Think of U_1 and U_2 as different frameworks from which to view an aspect of the world, rather like different units for measuring the same thing-feet and meters for measuring length. Although an actual U_1, electro-magnetism for example, has a history and may be convenient among equivalent frameworks, epistemologically it is an arbitrary choice, just as feet is an arbitrary choice of unit to measure length.

A more interesting way of comparing MTs is empirical equivalence, which we define as follows. Recall a predictive Procedure is a Procedure that has all its outcomes theorems or contradictions and all its executions have produced theorems. Define two MTs, U_1 and U_2, to be *empirically equivalent* if there is a translation between their underlying languages taking predictive Procedures to predictive Procedures in both directions. One can then have two MTs which are empirically equivalent but have contradictory axioms, a quite common occurrence.

Using empirical or strict MT equivalence we can now formulate notions of *viewpoint-free knowledge*, that is, absolute knowledge. Absolute empirical knowledge about some aspect of the world can be defined as an equivalence class of MTs with respect to empirical equivalence or, more intuitively, as an MT, keeping in mind that an empirically equivalent MT is an equally valid viewpoint. The aspect of the world an MT is applicable to is characterized by the MTs Procedures. Of course the importance attached to a Procedure or its MT can vary wildly from culture to culture. Analogous definitions and remarks can be made using strict MT equivalence but absolute empirical knowledge is probably the important variety. Although created by humans, in all of our experience of ordinary technical and scientific knowledge of the natural world, a Procedure being predictive is beyond human choice.

11. Meaning, Facts and Laws of Nature

Mathematics has a very simple semantics. First there are undefined words, for example if you start from Bernays-Gödel set theory: **no**t, **or** , **For all** , **x** , **y**… ∈ , and =. Then all other words are defined in terms of the undefined words and previously defined words. Of course one has some intuition about these words but one goes well beyond any simple notion of intuition. "Set" is a simple word, as in a "set of dishes". But intuition is stretched when thinking about a set representing the first inaccessible cardinal. Probably in some strict sense, in mathematics, meaning is completely precise while being completely meaningless.

A more structuralist tack for mathematics would be to say the undefined terms gain their meaning from their place in the model as a whole. This is the approach taken by mathematics over the last hundred years. In Euclidean Geometry, lines are those things determined by two points, points are where two lines cross. A whole number gains its meaning by being one of the whole numbers making up a system which in turn has various properties by virtue of being in larger systems. Instead of pondering the meaning of "a number" or "the number one", as was part of early approaches to the study of numbers, concentrate on setting down a set if relations between numbers, which characterizes the number system.

The MT concept provides a better opportunity to discuss meaning than does mathematics. The meaning of **apple** is surely related to the Procedure **x is an apple**, notwithstanding some Postmodern doctrines to the contrary. But the meaning of "apple" is only illuminated when **x is an apple** is executed, in which case attention is drawn to only one apple. One "gets the idea" of apple from a number of executions of the Procedure and from executions of other Procedures such as **x is an orange**. But giving a precise meaning to **apple** is much like the problem of inductive reasoning giving empirical truth. As an MT evolves, possibly confronting questions such as "Is this an apple?" new Procedures are added which refine the meaning of its words. Meaning for the sentences and words in the model of an MT comes from the lexicographic definitions of the model, that is, words defined in terms of other words, and from the Procedures imprecisely giving meaning to the undefined words, i.e. e., an execution of x **is a straight line** for **straight line**. Procedures illuminating lexicographically defined words, by way of example, are also vital. But complete precision of meaning is lacking and is probably undesirable as being too constraining.

An attractive way of viewing meaning is as a relation between sentences. We name this relation "refinement" and define it by sentence p' is a refinement p if p' is a plausible answer to the question, "What did you mean by saying p?". Usually there are several possible answers having different meanings from one another. As to precise meaning, possibly it involves an infinite regression as, going from signifier to signifier without ever reaching the signified. In practice of course, the process usual stops after a few steps when the questioner is satisfied.

Another meaning of "meaning" is an answer to a question asking the meaning of an event. An MT for which the execution of some of its Procedures provide a description of the event could be taken as its meaning in as much as it provides a context for viewing the event.

Philosophers sometimes say a proposition about the world is true if it corresponds, or is verified, by the facts! But what is a fact? We define a *fact to* be an equivalence class of pairs (U,s) where U is an MT and s is a sentence in U (A point of view and an assertion from that point of view), with respect to the following equivalence relation: Pairs (U,s) and (U',s') are equivalent if and only if, U and U'are empirically equivalent by a translation, translating s into s'. Furthermore, we require the equivalence class to contain a (U, s) in which s was produced by executing a Procedure of U. By the same maneuver one may define a "law of nature" to be an equivalence class of pairs with respect to strict equivalence and requiring s to be a theorem instead of the outcome of a Procedure

12. Concluding Comments

Few things passing for knowledge fail to be expressible in a language, fail to use the words "and", "or", "not", "implies" and "for all" satisfying the usual axioms, fail to have sentences designated as true, at least in some formal sense, or fail to recognize that contradictions among the true sentences is to be avoided. Contact with the world, the material, natural and human worlds, is at least realized through some system of communal naming of things in some moderately consist way; all agree, "This is an apple", "This painting is in a painterly style", possibly mediated through some notion of "concepts". In addition, the designation of being true may be constrained by the outcomes of human interactions with the world, what we described as predictive power. That is, an MT (model (formal language) – technology (procedures for interaction with the world.)) can be associated to most bit of knowledge in a reasonably unambiguous way capturing the features listed above and in consequence the properties of MTs carry over as properties of informal bodies of knowledge.

We have emphasized two main axes of variation of MTs, the extent to which an MT is constrained by its predictive Procedures, which varies between having no predictive Procedures to being fully constrained by them, that is, being a mature MT, and a second axis, the extent to which its predictive power is dependent on the communities culture. Although cultural dependence is easily assessed in many cases - the sun rises every morning - it can be very difficult in the social sciences. An interesting example of the difference between the humanities and science is the contrast between History and Astronomy, both of which are unable to modify or manipulate their subject matter.

Perhaps a useful category of knowledge generalizing the Platonic Form would be the Deductive Sciences, which would include the humanities and mathematics. A great deal of discussion goes on in this category involving mixtures of science and the humanities, for example, science and religion.

Nothing in this paper provides a plausible, satisfying notion of truth, assumed to exist by most people, such that, in the same sense, each of the following sentences is either true or false.

The sun rises each morning.

God exists.

Water can be made to boil by heating it.

Thou shall not bear false witness.

Bibliography

[1] Derrida,*La Voix et le Phenomene*,

[2] *Encyclopedic Dictionary of Mathematics*, The MIT Press, (1977)

[3] Thomas S. Kuhn, *The Structure of Scientific Revolutions*, University of Chicago Press(1962)

[4] Jean-Francois Lyotard, extracts from *The Postmodern Condition: a Report on Knowledge*(1979) University of Minnesota Press.

[5] Karl Popper, *The Logic of Scientific Discovery*, New York Basic Books, 1959

[6] Andrew Ross, editor, *Science Wars,, Duck University Press 1996*

[7] Richard Rorty, *Introduction, Objectivity, Relativism, and Truth*, (1991), Cambridge University Press.

[8] Alfred Tarski, *The Concept of Truth in Formalized Languages*, Logic, semantics, metamathematics; papers from 1923 to 1938, Translated by J. H. Woodger,Oxford, Clarendon Press, 1956.

[9] Ludwig Wittgenstein, *Tractatus Logico-Philosophicus*, Routledge and Kegan Paul Ltd (1922).

.

Brandeis University
Waltham MA, 02454
U.S.A.
Brown@brandeis.edu