On Knowledge
From a Mathematics Perspective

By

Edgar H. Brown Jr

2011
For Gail
Introduction

Over the last two hundred years there has been a tremendous expansion of science and its application to practical aspects of human life via engineering, agricultural science and medicine. This has lead to drastic transformations of political, social and religious life. Sorting out the conflicts arising from these changes is an ongoing affair for the professionals: philosophers, theologians, social scientist, religious leaders, politicians and lawyers, and for the general public. How should marriage be regulated, is there a God, is Darwin or the bible right, to what extend should the government be in involved in the economy, how should cloning be regulated, how old is the universe? The aim of this paper is to present some simple ways of looking at our notions of what constitutes knowledge and truth, ways that may be helpful in thinking about these issues.

A feature of the modern world is the recognition and, more or less, the acceptance of the fact that different societies often subscribe to quite different religions, moral rules, conceptions of the natural world and broad worldviews. Cultural dependence is less clear with respect to notions of what constitutes a body of knowledge and especially the meaning of its being true. For both the man on the street and the philosopher of knowledge, in most cases, the truth of a statement may be in dispute, but generally there is agreement that it is either true or false, in some generally accepted sense. Every day sentences such as "The color of that chair is blue.", "You are lying.", "I planted grass seed on my lawn and if I properly water it, most of the
seeds will sprout." are not problematic, in contrast to say, "God exists", "Man evolved from monkeys", "The weight of an electron in is $9.11 \times 10^{-28}$ grams", "Thou shall not murder" from the ten commandments, or more broadly, the contents of the bible.

For the man on the street, what “truth” really means is not problematic nor much thought about, but among the experts, philosophers, theologians, and to some extent scientists, it is admittedly unclear and worthy of study. A major branch of philosophy, epistemology, with a long history, is devoted to clarifying what constitutes knowledge and the meaning of truth. For the most part, the major works in this field set forth, in some deep sense, answers to what constitutes a body of knowledge and what it means for statements in it to be true. Although aware that philosophy concerns itself with the nature of knowledge and truth, most scientists do not take it very seriously, confident that for their work, a simple approach will suffice. In contrast, many theologians are deeply involved in questions of epistemology.

Up until quiet recently, cultural dependence of notions of truth has received very little attention. The present controversy concerning Darwinian evolution versus intelligent design assumes that one or the other is true, while it is at least conceivable that the two sides represent different cultures having quite different concepts of truth, an approach congenial with the recent intellectual development called postmodernism. Encouraged by Thomas Kuhn’s analysis of scientific knowledge in *The Structure of Scientific Revolutions in Science* [2], which suggests that truths in science are socially determined, postmodernist doctrines now holds that each body of knowledge has its own internally defined notion of truth or validity and in
consequence, the validity of doctrines cannot be compared. Some postmodernists assert that areas of knowledge, say physics, political science and literary criticism, have built into them their own notions of truth making nonsensical the assertion of the truth or falsity of something in one area from the standpoint of the other. Often an analysis of a particular subject matter is deemed to be merely an interpretation based on the realities of political power. A good example of the postmodern viewpoint is provided by Andrew Ross, editor of *Science Wars*, in his essay, “New Age Technicultures” in *Cultural Studies*:

In this paper [i.e., the distinction between authentic science and pseudoscience] it is worth drawing an analogy between the demarcation lines in science and the borders between hierarchical taste cultures—high, middlebrow, popular — that cultural critics and other experts involved in the business of culture have long had the vocation of supervising. In both cases, we find the same need for experts to police the borders with criteria of inclusion and exclusion. In the wake of Karl Popper’s influential work, falsifiability ([5]) is often put forward as a criterion for distinguishing between the truly scientific and pseudoscientific. But such a yardstick is no more objectively adequate and no less mythical a criterion than appeals to, say, aesthetic complexity have been in the history of cultural criticism. Falsifiability is a self-referential concept in science in as much as it appeals to those normative codes of science that favor objective authentication of evidence by a supposedly objective observer. In the same way “aesthetic complexity” only makes sense as a criterion of demarcation inasmuch as it refers to assumptions about
supposed objectivity of categories like the “aesthetic”
refereed by institutionally accredited judges of taste.

A related concern was expressed in C. P. Snow’s “Two
Cultures” lectures in which he lamented the practitioners of
Science and the Humanities being ignorant of the each
other’s subject.

Not surprisingly, postmodernist doctrines turn out to be
entirely in the humanities as is almost all of philosophy and
philosophy of science. More surprisingly, Kuhn’s analysis
of science, which might be thought to bridge the gap
between the humanities and science, is itself largely in the
humanities. With considerable justification, postmodernists,
interpret Kuhn as claiming that the acceptance of a science
theory from among competing theories is not decided by
experimental evidence (a claim we will argue, is patently
false.)

Recent developments in epistemology tend toward
discrediting common sense views and their technical
renderings, as well as the traditional Platonic viewpoints, in
favor of a more postmodern approach conceding value as
relevant, that is, true and false is constrained by good and
bad.

A major aim of this work is to point out that our
standard notions of knowledge and truth are highly cultural,
to suggest ways of taking this into account, and to apply this
analysis to a variety of questions. It will emerge that a great
deal of the machinery of human interaction with the world is
devoted to correcting for viewpoint dependence at very
basic levels and in consequence goes unnoticed.

Our analysis is applicable to “bodies of knowledge”
including literary criticism, physics, philosophy, astrology,
mathematics, psychology, history, art, medicine, religion,...
as well as subunits of these fields. We will view knowledge as broken up into fairly self-contained bodies of knowledge dealing with large or possibly small aspects of the world, including human activities in the broadest possible sense. Characterizing the structure of these bodies of knowledge will play a central role in our analysis. Our approach is to single out two major intertwined aspects of a body of knowledge, the logical reasoning that goes on in it, which we express as a “theory” set forth in a language in which the elementary rules of logical reasoning apply, for example the logic involved in one deducing from “All men are mortal and Socrates is a man” that “Socrates is mortal.” Second, that the language is related to the world by ways of interacting with the world, an idea to be made precise, but includes the notion that someone holding up an object for inspection and declaring “This is apple.” has major relevance to knowledge about apples.

To carry out our arguments we need a precise formulation of our two parts of knowledge, language-logic and interaction with the world. Furthermore we aim to have our formulation as separated as much as possible from last 2,500 years of western philosophy. The exposition is meant to offer a way of looking at knowledge that is reasonably unambiguous and applicable to both social knowledge and the sciences. The two interrelated aspects of a body of knowledge, how it interacts with the world and the logical reasoning it involves, both important aspects for most bodies of knowledge, but may nevertheless omit other important features, for example, emotional content and contributions to social organization and cohesion. More importantly, it is very much a departure from the western tradition. A major claim of the argument is that a body of knowledge, with its “theory” and “practice”; is analogous to a painting of some
scene seen through human eyes, with the sentences corresponding to brush strokes. Just as a brush stroke in isolation from the picture is meaningless, so is an individual sentence in isolation from its language-ways of interacting with the world meaningless and can in any sense be true aside from being logically true. For example, it offers a solution to the chronic problem in traditional epistemology, how we have knowledge of things other than ourselves and the operations of our own mind. The world kicks back when you kick it. Granting that knowledge is very subjective, an important question we address is how we correct for this subjectivity. Although our theory-practice is applicable to both physical and social knowledge, it clearly accounts for their substantial difference.

To address the expository challenge presented by most readers for whom, “True is true and what’s the problem?”, Chapter I gives a somewhat fanciful history of the development of western civilization’s concepts of knowledge and truth; it turns out to be quite complex.
Contents
Chapter I An Historical Fiction 10
Chapter II “History” Epilog 31
Chapter III Language and Logic 1 35
Chapter IV Language and Logic 2 42
Chapter V Units of Knowledge 48
Chapter VI Models of Units of Knowledge 51
Chapter VII Procedures 59
Chapter VIII Simple Examples of Models 62
Chapter IX Models for Science and Engineering 64
Chapter X Models for Mathematics 70
Chapter XI Models for Social Knowledge 73
Chapter XII Empirical Truth 83
Chapter XIII Formal Truth 85
Chapter XIV Types of Knowledge 87
Chapter XV The Language of a Model 89
Chapter XVI Relativity of Knowledge 94
Chapter XVII Meaning, Facts and Laws of Nature 98
Chapter XVIII Kuhn and Postmodernism 103
Chapter XIX Platonic and Thales Forms of Knowledge 110
Chapter XX Concluding Comments 115
Addendum I: Set Theory 119
Addendum II: Gödel’s Incompleteness Theorem 119
Bibliography 121
Chapter I
A Historical Fiction

The pre-Christian precursors of our views on the nature of knowledge came from the early civilizations of the Tigris and Euphrates, the Egyptians, the Hebrews, the Greeks and the Romans, the Greeks having the most influence on our philosophy of knowledge. We begin with the development of mathematics, historically the study of numbers and space, and a major theme running through our story.

A good share of the mathematics one learns in school, arithmetic, solving quadratic equations and systems of linear equations, calculating areas and volumes, and geometric constructions, were known to the Egyptians, Assyrians and Babylonians by the end of the second millennium BCE. This ancient mathematics consisted of recipes for performing calculations used in commerce, construction and astronomy, with very little explanation or argument as to why they gave correct or approximately correct answers (for example, the ratio of the circumference of a circle to its diameter). Geometry arose in the valley of the Nile to meet the practical need to restore the boundaries of agricultural land after the annual inundation of the Nile River.

By way of an example of this ancient knowledge we give a fanciful version of Egyptian plane geometry. Each year the Nile flooded and in receding covered the cultivated land with mud. Imagine returning to your home after massive floods have swept away all the buildings and in receding covered everything, save a few landmarks, with mud. You ask, “Where are the boundary lines of my property?” The Egyptians had recipes for working this out which might have gone something like the following: On the
landscape there are various rocky points not covered by the flood waters and given names. In addition the Egyptians had various tools with experts trained in their use; for example, long ropes marked with knots at unit length intervals. Perhaps the unit of length is called a yard and the knots are at one-yard intervals. The recipe for a particular piece of property might be as follows. Stretch the rope from a marker on the rocky point called Aka to a marker on rocky point Rau. Starting at Aka and counting knots, drive a stake in the ground at knot 146 and another at 210 marking out one side of the piece of property. Now bring forward an apparatus, probably provided by a priest, which consists of three highly decorated wooden rods joined to form a triangle with sides 3, 4 and 5 yards long. The corner where the 3 and 4-foot sides meet is placed at the second stake point and the 3 side is lined up with the 146 to 210 segment of the rope, making a right turn as one goes from 146 to 210. A third stake is now driven into the ground at the 4-5 corners. This might be called laying out an Isis angle or what we call a right angle. Now lay down the rope starting at the second stake, on a line with the third and, keeping the rope straight and taut, drive in a fourth stake at the 76th knot. Continuing in this way, possibly needing more complicated angles, the boundaries of the property would be laid out. Egyptian geometry consisted in recipes of this type, giving a form of our high school plane geometry. The reader is urged to take note, in this example, of the intimate relation between theory (the directions, perhaps in a manual) and interactions with the physical world.

If you buy a lot in Vermont and arrange for carpenters to build a summer home on it, they will use essentially the same technique as the Egyptians in laying out your lot and the planed location of your house. In your county
courthouse essentially such a recipe identifies your property in your deed and you may have called on experts, surveyors, to locate your lot lines from a recipe in much the same way as the Egyptians did.

The Greek philosopher and scientist, Thales of Miletus (640-546), who had knowledge of Egyptian geometry and Babylon astronomy, is credited with founding mathematics as a deductive science, that is, organizing mathematics around demonstrating by logical arguments the correctness of ones assertions and calculations. In addition he recognized the possibility that material phenomena conformed to discoverable laws, which in the next three centuries were demonstrated to be the case in a variety scientific discoveries. For example, Thales set forth and demonstrated the general angle, side, angle theorem for congruent triangles and, combined with the similar triangle theorem, used it to measure the distance of ships from shore. World War I photographs of artillery equipment show a device for measuring target distances essentially the same as used by Thales. Unlike the Egyptians who only had calculation procedures giving answers to practical problems, the Greek mathematicians went on to insist on knowing why such procedures worked, based on logical arguments. By 300 BCE the Greeks had worked out plane geometry (Euclidean geometry) as a deductive system, that is, first principals and logical deductions from them. The first principals, the postulates, often still used in high school geometry classes, were as follows:

1 One and only one straight line can be drawn between any two points. (Note that this captures the idea involved in the Egyptian stretching the rope from rock Aca to rock Ra. This can be done and there is only one position for the tightly stressed rope.)
2 A finite line can be extended infinitely in both directions.
3 A circle can be drawn with any center and any radius.
4 All the right angles are equal to each other.
5 Given a line and a point not on the line, only one line can be drawn through the point parallel to the line (not intersecting the given line.)

Starting from the postulates, taken as true, all the other theorems (the true statements) were proved to be true by repeated application of the reasoning, if A and A implies B are theorems or postulates, then B is a theorem.

Indulging our historical fiction, a funny thing happened on the way to the Agora postponing the expansion of experimental science (with notable exceptions) for 2,000 years. While walking to the Agora, Socrates (469-399) thinks of incorporating the deductive reasoning methods of Thales into his pedagogical techniques thus inventing the Socratic dialogue method by which one can derive the true nature of such things as “justice” and “beauty”. In this method, the teacher invites the student to define or explain a concept, say “justice”. The student complies and the teacher points out flaws in his offering, which the student attempts to correct, and back and forth it goes, converging on the real and true answer. An essential element in this process is logical reasoning. Plato (428-348), a student of Socrates takes up this technique going so far as to have inscribed over the porch of his Academy: “Let no one who is unacquainted with geometry enter here.” It was not because geometry was useful in everyday life that Plato so stressed its study but because geometry trained the mind in correct, abstract thinking. In addition to Euclidean Geometry, the portions of Greek thought that were most influential in the west were largely the works of Plato and Aristotle (384-322) on
morality, politics, government, beauty and the nature of knowledge.

Euclidean geometry had the very attractive feature that one could, with a very high degree of certainty, assert the truth of all its theorems, certainly a great improvement in persuasiveness compared to the speeches in the Athenian Assembly of Citizens. In addition, the human mind was directly in touch with reality without reliance on dubious knowledge obtained through the senses. One needn’t depend on the senses to know that for every pair of points in space, there is exactly one straight line through them. On the other hand, messing about with ropes and mud, building roads or plastering walls, is work for slaves. The result in Plato’s philosophy was to make Euclidean Geometry the prototype for proper knowledge, but separating it from its practical applications. Plato envisioned a deeply real world, independent of man, only very tenuously accessible through the senses but which the human mind could directly know (a point of view still very much in vogue). This real world contained perfect points, lines, regular solids, spheres and in addition, perfect justice and perfect systems of government, all of which could be accessed through the labors of philosopher kings in much the same way as one discovers and proves theorems in mathematics.

As well as knowledge of this “ideal” world, there was another category of knowledge, quite imperfect practical knowledge, such as how to make beer, or barrels or build bridges, which belonged to the lower classes or slaves. There was also taxonomy: the identification and classification of the various “ideal” forms such as the various forms of plants and animals which, although discovered by observation, lived in the same place as points and lines in space. These activities occupied a midway
position between knowledge modeled on Euclid and practical knowledge.

By the forth century BCE the Greeks had organized most of their mathematical knowledge in the eighteen books of Euclid's Elements. Plain Geometry is the main subject in these books, thus the "Euclidean" geometry we study in school. Unlike an Egyptian compilation of mathematical knowledge, which would consist in a collection of recipes for performing various calculations and practical constructions such as in our Egyptian land surveying example, Euclidean geometry organizes knowledge by setting down postulates, that is, clearly true statements, and then, starting from the postulates, deducing by logical reasoning, all the rest of geometric knowledge.

Actually Euclid’s work used various observations that did not follow from his postulates. For example, if a point on a line is inside a circle, then there is a point on the line outside the circle. Hilbert, in 1899, gave a complete set of axioms consisting 16 statements about points and lines. He also gave axioms for three-dimensional Euclidean geometry, which has twenty postulates specifying relations between points, (straight) lines and planes in space, such as, “Through two points, there is exactly one line." With a little experience dealing with solid objects, one can easily become comfortable with the concepts of point, line and plane out there in space and be convinced beyond doubt that the postulates are true. By logical reasoning one means something fairly simple. Reasoning of the form: From “All men are mortal.” and “Socrates is a man.” we deduce “Socrates is mortal.” and we know “Today is Monday or today is not Monday” is true. Reasoning such as this and a few variations suffice to characterize logical reasoning, which we will spell out in more detail in chapters III and IV.
By 300 BCE the Greeks had in place all the elements that make up the modern science-engineering paradigm. Combining a precise formulation of logic with Thales’ method, the main elements of modern science were in place. The researcher, on the basis of experience of the world (the Egyptian land surveying techniques), develops the concepts (point, line and plane) and constructs an abstract theory about them (Euclidean Geometry). The engineer utilizes this theory to formulate practical procedures, which he passes on to craftsmen as recipes for tasks. From the vantage point of the theory, additional theorems and applications are sought. We call this the Thales Form of knowledge. Recent scholarship ([9]) has discovered that the Greeks very successful utilized the Thales Form paradigm in a modern way in the period from the death of Alexander the Great (323BCE) to the Roman conquest of Greece (57BCE).

For the next two thousand years, deductive systems, that is, systems consisting of postulates produced by deep thinking and put forth as true, and statements logically deduced from them, was the prototype for all serious knowledge. We call this mode, the Platonic Form of knowledge and briefly trace its history and its various applications. The importance of logical deduction remains constant while the source and the grounds for the truth of the postulates vary. Curiously, this emphasis on deductive systems, with geometry the example, survived well into the 1950’s in the teaching of high school Euclidean Geometry with not a hint that it had any practical applications, not withstanding that abstract geometry combined with interaction with the world – measurements- together its applications to the physical world, ha no serious competitor as the most important scientific theory and accompanying engineering practices until the seventeenth century and
possibly not until the nineteenth. Euclidean Geometry remains one of the principal underpinnings of our technical society. All construction of buildings, bridges, automobiles, etc. all intimately involve geometry. Probably the reason for Plato’s neglect of the practical uses of geometry was that his Academy was training the ruling elite, focusing on morality, religion, politics and what constitutes a good life.

An essential element of science is direct observation and interaction with the world. Plato set forth a very different doctrine, to the effect that knowledge cannot be derived from the senses; real knowledge only has to do with concepts. The senses only deceive us; hence we should, in acquiring knowledge, ignore sense impressions and develop reason. This raises the problem of “universals”. If our knowledge is to consist of true assertions, then words such as ”man”, “God”, “soul”, “straight line”, “color” and “justice” should be the names of things. Plato addresses this problem by postulating two worlds, out there!, the world of the senses, which we call the material world, and the world of ideal objects and their relations. Euclidean Geometry lends itself very well to this point of view. It supplied wonderfully solid truth about the world; who could doubt the truth of theorems in geometry. With respect to its subject matter, we draw a straight line on the blackboard, but it is only a crude approximation of a real, ideal straight line out there. Thus the existence of the ideal objects of geometry lends support to postulating the existence, out in the world, of other ideal objects such as justice, moral laws and gods.

The many works of Aristotle, a student of Plato, included a formalization of logical reasoning. In codifying logical reasoning, he set down rules of inference (if p and p implies q are true, then q is true) and recognized the importance of axioms for logic and axioms for the subject at
hand (often called postulates), definitions of terms and the importance of giving logical arguments starting with the postulates.

Logic provides a straightforward way of verifying that a line of reasoning is correct. Given a collection of assumptions, say the postulates of Euclidean Geometry, it is usually very difficult to determine whether a given statement logically follows from the postulates, that is, is true if the postulates are true. But if someone comes forward with a line of reasoning, a proof that the statement follows from the assumptions, then it is fairly straightforward to check that the proof is correct; logic provides a step-by-step way for doing this. Some readers will remember in high school plane geometry the difficulty of constructing a proof (line down the middle of the paper) and the ease in checking its correctness. (In Chapter’s III and IV we give a detailed account of logical reasoning).

Aristotle emphasized that to use logic to explore a subject, one needs some true statements to start with, which he called the postulates for the subject. To get truth about the world using logical arguments, one needs truth inputs to get truth outputs. In Plato's system where does the truth inputs come from? Again geometry comes to the rescue. Just thinking about it, and possibly engaging in Socratic dialogue, one comes to know about those straight lines and points out there and that through two points there passes exactly one straight line, quite independently of your stretching strings from pole to pole to mark out the boundary of your garden. The emphasis is on deep thinking, convincing arguments, Socratic dialogue, rhetoric and sometimes, sacred texts or wisdom. Plato suggested that each of us has all knowledge, or perhaps all-important knowledge, already in our heads, ready to be extracted by
Socratic dialog. A history of western philosophy, including contemporary developments, could be organized around the search for these input truths.

Alfred North Whitehead, a widely influential twentieth century philosopher and mathematician, commented,

“The safest general characterization of the European philosophical tradition is that it consists of a series of footnotes to Plato”.

For the first fifteen hundred years in the present era, the degree of culture of a European tribe, community, or nation was the extent to which its people were literate and knowledgeable of Greek culture.

In our “history” we next turn to the period 100 BCE to 400 CE, with the Roman Empire and the rise of Christianity being the main influences. Rome was outstanding with respect to the portion of knowledge in the Thales Form that could be called practical: construction of roads, bridges, aqueducts, public buildings, military organizations and techniques. In the Platonic Form of knowledge it was outstanding with respect to law and government. In addition it preserved Greek art and philosophy. With regard to the creation of new philosophy and science, it was zero. The few Roman books on science were “Greek science for dummies”.

In this period and to present times, Christianity has had a close and enduring relation with the works of Plato and Aristotle. For the beginning two hundred years of the Christian era, the Christian Gospels and Greek and Hebrew doctrines were intertwined producing a variety of Christian sects with the works of Plato and Aristotle making substantial contributions. Variations of Aristotle’s notion of “substances” were applied to Jesus, God and the Holy Ghost, that is, are Jesus, God and the Holy Ghost one
substance or three substances, and if three in what order? These distinctions contributed to the delineation of the many early Christian sects. Constantine legalized Christianity in 316, made it the state religion in 380 and settled on God the father, Jesus the son and the Holy Ghost, all one substance (Nicene Creed).

From about 300 CE Imperial Rome’s government was split into western and eastern parts, sometimes each with its own Emperor. By 400 the western part covering what is now France, Germany, the Low Countries, England, and Spain, had begun collapsing into various independent entities. In the West a semblance of the Roman Empire continued organized around the Christian churches and monasteries. The Elite of the clergy were literate in Latin and Greek and incorporated parts of Greek thought into church doctrine.

The Roman Empire, ruled by emperors in direct succession to the ancient Roman emperors, continued in the more populous eastern part of the Empire of 200 CE covering Greece, the Balkans, Turkey, Syria, Jordan, Israel, Egypt and parts of Italy. (Calling this later Roman Empire the Byzantine Empire is a purely modern western convention.) It was also the richest and sophisticated of the divided Empire with a literate Greek speaking elite educated in the Greek and Latin classics. Manuscripts were copied, commentaries were written, some by women. A large number of the Greek classics later found their way from them to the West substantially contributing to the Renaissance. The Roman Empire, continuing under the Ottoman Turks after 1453, continued the transfer of Greek and Latin classics to the Moslem world. Possibly due to the Empires preoccupation with expanding into the west, from 800 to 1100 the intellectual center of the world was in central Asia around Samarqand in contact with the entire
Euro-Asian continent and largely the doing of Persians and Turkics. Starting around 750 AD, science flourished under the Abbasid caliphs of Baghdad. Like the Christians, the Moslems incorporated portions of Greek thought into their culture, principally in mathematics and medicine, which in turn came to the West via Al-Andalus (most of Spain.)

The Renaissance is too complicated for our history beyond the obvious, it's great contributions to architecture, art, politics, commerce, politics and a general transformation of the way the world is viewed.

Various ways of treating Christian doctrine in the Platonic Form emerged in the Middle Ages, for example, Neoplatonism and Scholasticism. Thomas Aquinas (1225-1274) synthesized Aristotle and Augustine and worked out Christian theology in the Platonic Form, converting theology, more or less, into the proving of theorems, including five proofs of the existence of God. Since then, Christianity, unlike most other religions, has been lumbered with the requirement that its doctrines be true in the same sense as two points determine a line and water boils at 212º F are true. It is probably a uniquely Christian idea that God can be approached or understood through intellectual analysis.

Plato’s ideal realm provided a place in the world for the Ten Commandments to exist as part of the fabric of the universe. Aristotle, in his Metaphysics, identified the taxonomic categories, the “ideal forms”, with the true underlying substance of things, which live in the same place as points and lines while the particular examples we see consist of this underlying substance together with its particular, accidental manifestation. Aquinas utilized these concepts to explain the transformation of wine into the blood of Christ in the Eucharist by saying the priest
transformed the substance while leaving the accident intact; it still looks like wine.

What is so important about the reasoning of Descartes: "I think, therefore I am." Not a very enlightening bit of wisdom. But being undeniably true, it could be taken as a postulate and a beginning for arguments about existence, for example that he exists and then that God exists. He finds knowledge gained through the senses unreliable and writes in *Discourse on Method*, “those long chains composed of very simple and easy reasonings, which geometers customarily use to arrive at their most difficult demonstrations, gave me to suppose that all the things which fall within the scope of human knowledge are interconnected in the same way”. Thus he is viewing knowledge in the Platonic Form. Since the material world makes itself known to us through the senses, an unreliable source, Descartes decides that there are two kinds of substance, material substance and thought substance. To jibe with "I think, therefore I am.", man is made of both kinds of substance. The thought kind also provides a home for God, the Ten Commandments and probably for Plato’s ideal forms. The downside of this conception is that there seems to be no way for the material and thought substances to interact, thus giving rise to the mind-body problem that has plagued psychology well into the last century.

Spinoza enrages both the Christian and Jewish communities by arguing that one doesn't need God to explain morality. Much influenced by Descartes he writes up his ethics as if it were a mathematical text, with postulates, definitions, lemmas and theorems.

Newton titles chapter II in his Principia, “The Laws of Motion, the Axioms” and sets forth his work entirely in the Platonic Form. When Thomas Jefferson, writing the
Declaration of Independence says, "We hold these truths to be self evident" he is following the Platonic Form; he is announcing that he is about to set forth the axioms from which his conclusions will follow. The eighteenth century doctrine of natural rights is a search for Euclidean postulates for politics. Hegel’s dialectical idealism only makes sense if there is some ideal realm (ideal in the sense of ideas rather than perfection) in the universe in which the dialectic may take place. Jesuit theology is dominated by the deductive approach and modern continental philosophy is incomprehensible without a knowledge of Plato, Aquinas and Aristotle's metaphysics.

During the Middle Ages the most accepted view was that religion provided the basis for morality and for social structure. God, in creating the world, created The Great Chain of Being: God, Seraphim … Angel, King, Prince, Duke, … Knight, peasants, an arrangement of animals and an arrangement of minerals. Beginning more or less with Descartes, western thought moved away from this to Plato’s idealism epistemology with the foundations based on deep thought on the lines of Descartes approach. Some of the major variations in this line of thought include the works Spinoza (1624-1677), Berkeley (1685-1753), Kant (1724-1804), Hegel (1770-1831), Marx (1818-1883), Heidegger (1889-1976) and Husserl (1859-1938).

With the rise of experimental science, the Platonic Form for knowledge came under increasing attack and from the sixteenth century various modifications have been proposed. British empiricism voiced great skepticism about the possibility of knowing the truth of a statement, for example a postulate, without reference or checking against experience. Francis Bacon, in his book New Atlantis (1627), proposes a new approach to knowledge. Through careful
observation, experimentation and inductive reasoning, man could discover the "laws of nature" which would replace divining them by pure thought. The goal of acquiring knowledge was the good of mankind, to be realized through applied science, in contrast to knowledge being restricted to religion and ethics or being something to be contemplated by proper gentlemen (as Plato recommends). This was to be an ongoing process instead of a big system to be set up once and for all. The Baconian viewpoint is shared by most contemporary scientists and might reasonably be called the Standard Science Theory of Knowledge. Unfortunately it suffers from some serious difficulties, and a large share of epistemology from the sixteenth century on has consisted in reformulating this Standard Science Theory by tinkering with the Platonic Form. Soon after Bacon, Locke and Hume were arguing that all knowledge must ultimately rest on sensory perception. As did the Greek aristocrats, these proper English gentlemen gave no attention to mucking about with mud and ropes, musing instead on what it means to say, "That chair is green." is true. Then Berkeley went completely over the top arguing that to perceive is to be and to be is to be perceived. If no one sees a tree fall in the forest, does it happen? Fortunately God is there to see it happen. The point is that if you become too dependent on sensory perception for knowledge of the world there may be no place for God or the Ten Commandments.

Inductive reasoning has been a perennial problem for the Standard Science Theory of knowledge. It consists in the process of passing from a number of observed cases to a general statement. For example, suppose there are 41 recorded cases of people eating those cute red mushrooms with the white dots and getting very ill and dying. On the basis of the 41 cases, can one conclude that the red
mushrooms with the white dots are very poisonous? Nowhere in anyone’s plausible logic can one make that jump. The fact that the sun rose every morning for the last thousand years does not allow one to conclude that it will rise tomorrow morning. Witness the drips from a dripping keg of wine. An immense amount of agonizing has gone on over this. We will argue that the change from deductive to inductive reasoning is the wrong tinkering with the Platonic Form.

Ironically, Euclidean geometry has been a plague on epistemology. Every theory put forward had to allow, or better still, account for the truth of Euclidean geometry, a serious burden. Whatever our epistemology, surely two points determine a line. In addition, it had the unfortunate effect of encouraging the notion that one could know some things with absolute certainty. Toward the end of the eighteenth century, Kant explained the validity of the Euclidean postulates by saying man does not merely take in raw sensory data; in taking it in, he processes and organizes it as built-in a’ priori truths. The postulates are part of his organizing machinery for perceiving space, which is why they are so clearly true (and give examples of a’ priori truths). This was quite a nice solution, but it came unglued in about thirty years due to the discovery of non-Euclidean geometries. One of the postulates for plane geometry (Euclid’s famous fifth postulate) states that given a line and a point not on the line, there is exactly one line through the point parallel to the given line, that is, not intersecting the given line. There was long the feeling that perhaps the human mind could not be certain of its truth but that it should be true (the system does not work well without it) so it must be a theorem. All attempts to prove it failed and finally the mathematician Lobachevski proved to everyone’s
satisfaction that it was not a theorem. He produced a mathematical model, that is, two collections of things called lines and points, specifies when a point is on a line and are interrelated so as to satisfy all of Euclid’s postulates except the fifth and hence raised the possibility that our real world space is not Euclidean. The following is such a model: Fix a circle in a plane. Take as all the points, the points inside this circle. Take as lines the portions inside the fixed circle of circles meeting the fixed circle at right angles. On easily verifies that this points and lines set up satisfies Euclid’s postulates except the parallel axiom, from a picture that there are an infinite number of lines through a given point and not intersecting a given line.

Lobachevski showed his result to Gauss who was one of those really irritating people who, when you show them your discovery, says oh yes, I knew that twenty years ago. A bit of mythology is that Gauss, a contemporary of the powerful Kant who had explained how a’ priori we known Euclidean Geometry, decided that it was best to leave his paper on the existence of non-Euclidean geometries in his desk drawer. Gauss attempted by an experiment to determine whether our space is one or the other by measuring the sum of the angles of a very large triangle formed by three mountain peaks. In Lobachevskian space this sum is less than 180º. Within experimental error, Gauss obtained 180º, making his results inconclusive. (The difference between Euclidean and a Lobachevskian space measurements could be very small.) Perhaps on an astronomical scale there might be a significant departure from Euclidean geometry.) With advances in physics and the fifth postulate debacle, the Platonic Form faded away as a serious contender for an epistemology for science.
The next development was Logical Positivism, in a way, an attempt to combine the Platonic Form with British empiricism. Logical reasoning is an important part of most people's notion of “knowledge”, but logical reasoning only produces results of the form, if such and such is true, then something else is true. One needs “true” statements to start. The Platonic Form in one way or another is based on the existence of some special realm, for example, Plato’s ideal objects, which man could know directly and which could provide a starting point. Logical Positivism is a twentieth century variant of this approach which, in its simplest form, takes as postulates all statements of direct observation — “That chair is blue”, “The volt meter is reading 2.4 volts.” A difficulty with this approach is you cannot produce any universally true statements, “All men are mortal.” for example (again the problem of inductive reasoning.) In Logic you need “For all” statements in to get “For all” statements out.

Wittgenstein set forth a very influential version of this in his book, *Tractatus Logico-Philosophicus*. According to the *Tractatus*, the world is the totality of irreducible facts and their interrelationships. Elementary propositions (the axioms) picture these interrelationships much as a map pictures the features of a portion of the earth's surface. The logic is then a reflection of the way the world is put together. Unfortunately, very little “knowledge” can be generated this way. By a very thin argument, he then concludes that any purported proposition not generated in this way is nonsense, thus eliminating propositions setting forth traditional philosophy, ethics, aesthetics, religion and, as he concedes, the content of the *Tractates*. Although he does not say so, presumably the “laws of physics” are also nonsense. In a subsequent work, *Philosophical
Investigations, he abandons this draconian position. In a watered down version of Plato he substitutes concepts for ideal objects, abstract nouns being the names of “concepts”. Thus “red” is the name of the concept red whose meaning cannot be explained but only shown by pointing to it.

A more elaborate form of Logical Positivism was created by Carnap in which he adds, in a complicated collection of ideas involving probability and "all possible worlds", a notion of degree of conformation, instead of truth, for sentences. These efforts went on in an intellectual climate of empiricism which, more or less, held that true sentences could be divided into two types: analytically (logically) true as in "Today is Wednesday or Today is not Wednesday" and empirically true, "Harry Truman was the President of the United States.". By some example sentences, Quine showed that this distinction did not make sense, the distinction being in the eyes of the beholder. In any event, Logical Positivism has greatly faded. The latest word, enunciated by Rorty and called pragmatism, is that any serious intellectual effort may be valuable. The main basis for his argument is that the Logical Positivism-Empiricism analysis of science and of knowledge in general is seriously flawed; for the non-philosophers, not a very compelling basis from which to argue.

The Greeks, in third and fourth centuries BCE, speculated about the nature of the world and the nature of knowledge of the world and in the process establishing the two main strains of the West’s theories of knowledge. One view was that the world was made of matter, of various types, and man’s knowledge of it is gained through experience or simply via his senses, sight, hearing, smell and touch. The other view was that the world consists of ideas (also called “forms”), “man”, “justice”, “love”, “gold”,

28
“heaven”… . Plato’s view was that the important parts of real world consist of such ideas, which he takes to exist independently of man and outside of space and time, not remarkable in a world that included nonhuman, thinking beings. Man’s knowledge of these ideal objects is gained by deep thinking and logical reasoning. In philosophy these two approaches to the foundation of knowledge are now called, respectively, empiricism and idealism. Variants of this pair are science and religion, earth and heaven. Actually, from the first century BCE to seventeenth century CE, Idealism was the only intellectual game in play.

From the many important Greek philosophers, in our “history”, we have singled out Plato and Aristotle due to their consistent importance in western abstract thought about what constitutes knowledge and truth. Up until modern times, important knowledge was as Plato specified, that is, as consisting of true basic principles obtained by deep thinking, and logical deductions from these principles. The existence of true statements produced by deep thinking was universally accepted and for most of this period “God exists” was one of these true basic principles or a theorem.

In some cases, postulates are quite explicit, for example, looking ahead to Descartes, “I think therefore I am.” is his starting postulate. More often postulates are not explicit, being replaced by manipulating concepts. For example, the ontological proof of the existence of God goes as follows:

The essence of anything consists in the collection of qualities it possesses. A thing or being is inferior to another being having the same qualities plus the quality of existing. Hence if God did not exist, there would be an essence superior to him having his qualities plus existing. Therefore being the most perfect being, he must have the quality of
existing. Hence God exists. Wow! (The argument depends on implicit postulates, for example “existing is better”.)

The Platonic Form of knowledge, the combination of Aristotle's logic and Plato's ideal world, more or less gives idealism and is the prototype of the underlying epistemology of the humanities to the present day, with some exceptions, e.g. Derrida who calls it logocentrism (Our treatment of meaning is vaguely similar to his différance.) The subsequent course of Plato’s approach occupies the core of a sequence of not very drastic variations of Plato’s “truth imputes”, and in Kant’s sense, a' priori knowledge about an ideal world. Kant was, as were the above philosophers, confident that man could discover knowledge of the world not depending on experience of the world, a' priori (pure of empirical knowledge), obtained via "pure reason".

Over the last 2,300 years, Plato’s “ideal world” goes through a variety of transformations, various aspects of consciousness being a popular contemporary substitute. For example, after 2,300 years, Husserl (1859-1938) is marvelously Platonic in his expectation that transcendental-phenomenological-consciousness will be able to “intuit the essences of objects and to thereby establish the universal structures of human consciousness providing the grounds for procuring absolutely certain and valid knowledge about things and events.
Chapter II
“History” Epilog
(Did Plato get it right?)

The main theme of our “history” of the West’s conception of knowledge was that a body of knowledge consists of a logical system along the lines of high school plane geometry, with the underlying axioms coming from a variety of sources. For Plato and most of his followers, axioms came, in one way or another, from deep thinking. These efforts were largely devoted to setting forth a basis for what might be called social knowledge, morality, religion, politics, government and what constitutes a good life. Religion’s axioms came from scriptures and deep thinking, a notable example being the works of Aquinas. Non-religious approaches divided into pure deep thinking, for example Kant, and empiricism, i.e. Lock and Hume. British empiricism envisions knowledge of the world starting with imputes through our senses — sight, sound, hearing and touch. Bacon envisioned aggressive careful recording of detailed descriptions of the world (providing axioms) from which general laws could be deduced (how?).

In the West between 400 and 1500 the Christian clergy were largely in charge of intellectual life, initially opposed Greek thought as pagan, but ultimately incorporating Plato into their doctrines and Aristotle’s metaphysics and logic into the Catholic Church’s theology.

A second, minor theme in our history is science, briefly flowering in ancient Greece and, beginning at the end of the 17th century, exploding into modern times. Although Aristotle wrote voluminously on many topics including science, his formulation of science had no significant
influence on subsequent science. English empiricism was influential in philosophy but its influence on subsequent science is at best ambiguous. So far, the rise of science has produced only minor alterations to the accepted Platonic Form. For example, the public and to a lesser extent scientists, in their view of science, very much over emphasize theoretical “laws of nature” in contrast to experimental techniques and results. In the discussions of the “laws” of Creative Design and of Darwinism, the empirical consequences of one or the other seems to play no role in the discussion; they tend to be viewed as potentially Platonic abstract laws.

A major claim of this work is that from the vantage point of modern science, Plato and his successors’ view of the nature of knowledge, as applied to science, is very wrong and possibly also wrong for social knowledge. Knowledge about the world is not just in our minds, nor in deep thinking, nor in language, nor consciousness, nor based on a’ priori truth. More obvious in modern science, but always clear, a central feature of human knowledge of the world critically involves humanly constructed interactions with the world and the reporting of the outcome of these interactions in a very disciplined way. Experiments in science often involve experts’ delicate use of complicated man-made equipment, which has been in development over many years. Similarly, in ordinary life, the technology that can produce the sentence “The color of that chair is blue.”, is extraordinarily complex and has been developed over millions of years, or created by God.

There is some notion that apples are real objects, in contrast to electrons, which are constructs of physicists. If one asked an experimental physicist about electrons, he might respond by rummaging in his desk to bring forth a
photograph with a black background and a lot of white lines and curves emanating from a single point, indicate one of the lines and say, "There's the path of an electron." To the question, "Does that electron have the same reality as that apple on your desk", he might respond with, “The apple and the electron share the theoretical machinery that goes into the concept of "thingness" and the electron needs a more elaborate theoretical framework, but the detector used to see the electron is childishly simple compared to the detector needed for seeing an apple, namely, our brains and eyes. (By way of perspective, humans, when invited to think about the world out there tend to look around their surroundings, the room, the outside, the people out there. They “see” it through their eyes; humans have about 300,000 sensors in their eyes and 30,000 in their noses. Dogs have the numbers reversed; do dogs “see” the world through their noses?) Retuning to apples and electron, someone might respond with, "But it looks like an apple!"

We make him stand in the corner. To put these two concepts, apple and electron, on a more equal footing think of two robots designed, respectively, to detect apples and electrons. A robot has a computer as brain, hard drive being its memory and the processor and software its thinking and operating machinery, and various sensing devices and hands, arms and legs by which it interacts with its surroundings. Is there any basic difference between the two robots? The picture of the robot is quite telling; there it is, interacting with the world, acting on the world and receiving inputs to its detectors from the world and forming models of the world on its hard drive. Better still; picture a group of robots interacting with the world including each other.
For our purposes, a clear concise formulation of logic and logical reasoning is essential. These are the topics of the next two chapters.
Logical argument played a central role in our history of the west’s view of knowledge; it was a major element in Socratic dialogue, in Euclid’s Elements and it plays an important role in subsequent ways of dealing with knowledge. Both language and logic comes with massive philosophical baggage. A major part of Western philosophy is devoted to various approaches to understanding the relation between language-logic and the world. Being involved with “truth”, logic has a long and rich history and is often imbedded in various philosophical works. In the past a great deal of agonizing went on about what the subject of logic should be, in particular should it be about “propositions”, mysterious sentence like objects which correspond to states of the world. One wishes to talk about statements such as: “Silver is a metal.” or “My necktie is red.”, that is, states of the world. But one cannot merely settle for states of the world because one must give attention to “My necktie is red.” even if my necktie is blue, that is, when “My necktie is red.” is false. One of the many approaches was to make a proposition an abstract entity about which one may have a belief. Fortunately this was much dephilosophied in the nineteenth century.

Aristotle attempted to reduce all logical reasoning to repeated applications of some simple lines of argument, which he called syllogisms. The following are examples:

All men are mortal and Socrates is a man. Hence Socrates is mortal.
Some men are married and all married men have spouses. Hence, some men have spouses.

He gave six more similar lines of reasoning and claimed that all logical reasoning could be reduced to a sequence of syllogisms. Up until the nineteenth century, his analysis was taken to be the last word on logical reasoning. This was unfortunate in that he muddled up the relation between “all men” and “man” and could not deal with sentences such as “Nobody knows everybody.” or “Any schoolchild can master any language.” Also, Aristotle’s recipes for logical arguments were not followed in Euclid’s Elements.

Fortunately for our purposes logic has been separated from philosophical complexity by two major developments around 1900. First, Aristotle’s mistakes were corrected by Frege (1848-1925) who sorted out the logical status of the relation between “All men” and “is a man”, replacing “All men are mortal” with, roughly, “If something is a man, then that something is mortal.” This is the main subject of chapter IV, “Language and Logic Part 2”. It turns out to be somewhat complex and technical, requiring a new grammatical noun category and requires only one axiom instead of Aristotle's six forms of argument.

The second big development was a gradual separation of semantics (meaning) and syntax (grammar) and the realization that logical arguments only depend on syntax. This led to the notion of formal languages, no meaning to the words, just grammatical type: verb, noun, predict, etc, which provides the present broadly accepted formulation of logic and the logical foundation of modern mathematics. Curiously, the subject of formal languages is one of the major intellectual developments of the late nineteenth and early twentieth centuries but has had little public attention or
popular accounts; it is important in modern logic, for example in Gödel’s work on incompleteness, in coding, in Chomsky’s linguistics and in computer science. In chapter VI, as an approach to analyzing bodies of knowledge, we will associate to a body of knowledge a formal language that captures its logical structure - its theory- and its predicates providing names for its ways of interacting with the world.

Logic gives a precise way of validating lines of reasoning, of judging a logical argument as to its correctness. In plane geometry, sometime in the past, someone thought up the theorem that asserts the equality of interior angles in parallel lines cut by a transversal, saw that it might be true, might be useful in thinking up additional truths and may have important practical applications in surveying. Presented with a proposed proof of this result, logic provides a way of definitively deciding its validity, but logic contributes very little to the actual discovery of this theorem. Within a formal language, logic gives a straightforward, basically mechanical procedure for checking whether a proposed proof is correct.

Recall, words and parts of sentences, are symbols, “gold” is the name of a symbol not a metal, the predicate of “John jumped over the curb” is “jumped over the curb”. A formal language can be approximately constructed as follows; start with some text, say a chapter in a textbook, and put quotation marks around each noun and predicate. The sentences then become sequences of symbols: the symbols “today”, “is warm” may be combined to make the (formal) sentence “today”“is warm”, two symbols set down in a row. Classifying the symbols as to grammatical type, noun verb, etc together with rules of grammar enables one to construct formal sentences. One winds up with a collection of symbols and rules for putting them together to form
sequences of symbols called formal sentences. Actually, to
find a set of grammatical rules accurately characterizing all
sentences for a natural language such as English is very
difficult, as Chomsky discovered.

To be less clumsy and preserve quotation marks for all
of their varied uses, we hereafter use boldface instead of
quotation marks for names of symbols in our formal
languages: thus today and is warm are symbols that can be
combined to make the (formal) sentence today is warm. We
very much simplify our grammar by restricting grammar
types to predicates and nouns, with nouns divided into
proper nouns, pronouns (common nouns are converted to
predicates, “book” becomes is a book) and a new type of
noun called a variable (see the next chapter). In contrast
with nouns, the concept of predicate in English grammar is
quite imprecise and is usually explained using example
sentences. We define a predicate to be an English sentence
with one or more nouns removed, as “is happy” is a
predicate from “John is happy.” and, as will come clearer,
“loves” is a predicate from “John loves Mary” (later we will
call these word combinations prepredicates instead of
predicates.) To simplify our formal languages’ grammar we
use sequences of names of words that can easily be
identified as nouns, predicates and logic words. For
example, and is a logic word, John, John Smith, it are
nouns and is tall is a predicate and they can be combined to
make (formal) sentences: John is tall, John Smith is tall, it
is tall and John Smith is tall and it is tall. The actual
words used to make John is tall act as mnemonic devices
which helps keep track of the grammar and if it is built out
of the sentences in a body of knowledge, allows one to carry
logical arguments in the formal language back to arguments
in its body of knowledge by removing the bold face.
To begin logic we designate **and**, **or**, **not** and **implies** as logic words and rules of grammar for their use as follows: Any pair of sentences, say **today is warm** and **tomorrow will be nice**, can be combined to produce **today is warm and tomorrow will be nice**, that is, a sentence, followed by **and** followed by a sentence, and similarly for the symbols **or** and **implies**. The symbol **not** is combined with a sentence by preceding it: **not today is warm**, which we specify as becoming the sentence “Today is not warm.” when the bold face is removed. These rules may be better said as follows.

If $p$ and $q$ are sentences in a formal language, $p \text{ and } q$, $p \text{ or } q$, **not** $p$ and $p \text{ implies } q$ are sentences. Note these constructions can be repeated and parentheses are included as needed: $(p \text{ and } q) \text{ implies } q$ in contrast to $p \text{ and } (q \text{ implies } q)$. Removing bold face from **today is warm implies tomorrow will be nice** may be rendered as “Today is warm implies tomorrow will be nice.”, or as, “If today is warm, then tomorrow will be nice.”

Some sentences in a formal language become (logically) true when the bold face is removed. For example, for any formal sentences $p$ and $q$, all sentences of the form $p \text{ or } not p$ and all of the form $(p \text{ and } q) \text{ implies } q$ become true when boldface is remove. There are a lot of sentences of this type, true by virtue of the above usual meaning of the above logic words. They and their formal versions are called **tautologies**. There is a simple tedious way of testing whether a sentence is a tautology. Specifying the standard meanings, $p \text{ and } q$ is true if $p$ is true and $q$ is true and false otherwise, $p \text{ or } q$ is true if $p$ is true or $q$ is true or both are true and false otherwise, $p \text{ implies } q$ is false if $p$ is true and $q$ is false and true otherwise and **not** $p$ is true if $p$ is false and true if $p$ is false. To verify that $(p \text{ and } q) \text{ implies } q$ is a tautology, try all possible combinations of true and false for $p$ and $q$ and
see if you always get true. If you do, it is a tautology and this method works for any tautology. (Using T and F for true and false and taking p and q as F and T, (p and q) implies q becomes (F and T) implies T, (F and T) is F, hence it becomes F implies T, which is T. The reader might try the above for p and q as F and F, T and F, T and T, verifying that all sentences of the form (p and q) implies q are T.)

In chapter IV we add logic symbols ∀ and ∃, which become “for all” and “for some” when the bold face is removed (∃ may also be read as “there exist”) completing our notion of a formal language and their logically true sentences. Roughly speaking, to deal with “All men are mortal.” “he”, “she” and “it” are combined into one thingness pronoun, it, and “All men are mortal.” becomes “For all it, if it is a man, then it is mortal.”, or formally, ∀it if it is a man , then it is mortal, where it is acting as a pronoun, thus finessing the logical relation between “all Men” and “man” (It is best not to get too tense about quotation marks. No serious language is unambiguous and can always be refined.) Extending logical truth to sentences evolving ∀ and ∃ uses the approach of Euclidean geometry: axioms, rules of inference, and the theorems (logically true sentences) with proofs.

The definition of “predicate” is extended to sentences with two or more nouns removed (sometimes called relations) such as the predicate in “John and James are brothers”, that is, _ and _ are brothers, which is called a 2-ary predicate. Putting proper nouns in the blank spaces gives a formal sentence John and Bill are brothers. Similarly _ and _ are son and daughter of _ is a 3-ary and an n-ary if it takes the insertion of n proper nouns to convert it into a sentence. John and James are brothers, John and Ruth
are son and daughter of Judith, and Ruth is happy are sentences and can also be described as 0-ary predicates.

Briefly, in chapter IV we complete our account of logic as follows: Following Russell and Whitehead’s *Principia Mathematica*, we add another type of noun, x, y, z, ..., called variables, replacing the *it* above and add logical symbols ∀, and ∃ for “for all” and “for some”. We then redefine formal language to be a collection of predicates, nouns and the logic words, rules of grammar, logic axioms involving the logic words and give rules of inference by which theorems can be deduced from axioms. A formal theory is defined to be a formal language together with subject matter axioms (as in Euclidean Geometry) involving the nouns and predicates of the formal language and providing theorems in the formal theory, as specified by the rules of inference: axioms are theorems and if p and p implies q are theorems, then q is a theorem.

In chapter IV we go over all of the above ∀ and ∃ material in more detail.
Chapter IV
Language and Logic 2

Recall a formal language as described in chapter III, Language and Logic 1, included a collection of symbols classified as nouns and predicates, all in bold face type, and logic symbols, and, or, not and implies together with rules for combining these symbols to form sentences. In addition, machinery is provided by which some logically true sentences, tautologies, are formally specified. Removing bold face converts formal sentences into (clumsy) English sentences and tautologies into customarily logically true sentences.

As outlined at the end of chapter III, we need to expand our notion of formal languages to include a formal version of sentences such as “All men are mortal.” appropriately related to is a man and is mortal. Frege’s idea was to replace “All men are mortal.” with “He is a man implies he is mortal.”. It works out very well extending Aristotle’s formulation of logic and replacing his syllogisms with one simple axiom. Clearly the pronoun “he” is too restricted. “Anything that is a man implies that thing is mortal.” is better. We use what Frege, more or less used, “If x is a man then x is mortal” where “x” is sort of a noun.

Descartes was the first to use x as the number to be discovered, as in a word problem: “Let x be the speed of the second train.” In Descartes’ case, “x” is an ordinary English language pronoun, in a context, naming a specific thing, quite different from the x in “If x is a man then x is mortal.” where what is meant is “For all things x, if x is a man then x is mortal.” In eight grade mathematics the difference between “x” in “x is the speed of the second train.” in a
word problem, and

\[(x+1)(x-1) = (x^2 - 1)\]

meaning for all numbers \(x\), glosses over the same distinction. For Frege, “If \(x\) is a man then \(x\) is mortal.” is not a sentence. “For all \(x\), if \(x\) is a man then \(x\) is mortal.” is a sentence. Also he specifies “For some \(x\), if \(x\) is a man then \(x\) is mortal.” is a sentence. Frege’s notation for all of this is complicated. The above and below is the notation in Russell and Whitehead’s Principia Mathematica [9].

To formalize the above we add to formal languages, \(x, y, z\ldots\), and logical symbols \(\forall\) and \(\exists\), which become “for all” and “for some” when the bold face is removed (“for some” may also be read as “there exists an”). Also add parentheses \((\text{ and } )\) to be used as needed. The symbol \(x\) acts grammatically somewhat like a noun but not entirely; \(\exists x\ x \text{ is a man}\) is a sentence but \(x \text{ is a man}\) is not. The standard terminology is to call \(x\) a variable and combinations such as \(x \text{ is a man}\) and \(\forall x\ x \text{ is a man and y is happy}\) are called formulas. In this language \(x\) is a bound variable and \(y\) is a free variable in the formula \(\forall x\ x \text{ is a man and y is happy}\) (italics indicate a definition is being made). In keeping with our using ordinary English grammar terminology we replace the word “formula” with “predicate”. We call \(x, y, z\ldots\) variables and classify them as nouns and modify which symbols are predicates as follows: is man and loves are renamed prepredicates; \(x \text{ is man}\) and \(x \text{ loves y}\) are basic predicates (This makes clear the distinction between \(x \text{ loves y}\) and \(x \text{ loves x}\)). The notion of basic predicate is elaborated bellow. To precisely define which sequences of symbols form sentences and which form predicates is reasonable straightforward but long and tedious. We try to get across the main points by examples:

\[(\forall x\ x \text{ is man and x loves y}) \text{ implies } z \text{ loves y}\]
is a \textit{predicate} as is any combination that becomes an English sentence when bold face is removed: “For all x, if x is a man and x loves y, then z loves y.” In this predicate, x is a \textit{bound variable} because of the presence of \( \forall x \) (or \( \exists x \)), and y and z are \textit{free variables}. It is a 2-\textit{ary} predicate because it has two free variables. If a predicate has n free variables it is an \( n \)-\textit{ary} predicate.

\((x \text{ is man and } x \text{ loves } y) \text{ implies } z \text{ loves } y\) is a \textit{basic predicate} because it does not contain \( \forall \) or \( \exists \). When we associate a formal language to a body of knowledge, its ways of interacting with the world are named by basic predicates and when a basic predicate is executed in the world, the outcome is a sentence with the predicate’s variables replaced by proper nouns and possibly with \textit{not} in front. In response to “Is Bill happy?”, executing the procedure \( x \text{ is happy} \) could produce \textit{Bill is happy} or \textit{not Bill is happy}. In contrast, a typical “law” is a sentence containing \( \forall \); \( \forall x \text{ x is a man implies x is mortal} \).

The following are sentences (no free variables).

\( \forall x \text{ is a man implies x is mortal} \)
\textit{not(George Eliot is a man)}
\( \exists x \exists y \text{(George Eliot loves x and hates y)} \)

To accommodate Frege’s program we also have to modify what happens when bold face is applied to common nouns (nouns that do not name unique things, such as “water”). For example, with respect to the noun “water”, the sentence “Water is a liquid” should become \( \forall x \text{ x is water implies x is a liquid} \). In passing to bold face, convert a common noun to a predicate: silver becomes \textit{is silver}, president becomes \textit{is a president}. Applying “bold face” requires judgment.

It has been proved that there is no algorithmic test (routine procedure) that captures all logical truths involving
∀ and ∃ as there is for tautologies. Logical truths, that is, logical theorems, are specified by axioms and the usual rule of inference that axioms are theorems and if P and P implies Q are theorems then Q is a theorem.

The remarkably simple axioms for ∀ and ∃ are (more or less):

If P(x) is a predicate with one free variable x and A is a proper noun, then

∀x P(x) implies P(A) (that is, If for all x, P(x) is true, then P(A) is true) and

P(A) implies ∃x P(x) (If P(A) is true, then for some x, P(x) is true.) This axiom can be eliminated by making ∃x an abbreviation for not ∀x not (It is not true that for all x, P(x) is not true), in which case this axiom becomes a theorem.

One can also derive all tautologies from axioms, for example the following (probably) suffice for the axioms:

p implies (q implies p)

(not p implies not q) implies (p implies q)

(p implies (q implies r) implies ((q implies p) implies (p implies r))

Sorting out precisely the notions of nouns, predicates, sentences, variables, axioms, theorems and logical systems emerged in its present form from research over a forty year period around 1900.

We define a formal theory to be a formal language together with subject matter axioms (as in Euclidean Geometry) involving the nouns and predicates of the formal language and providing theorems in the theory, as proved by rules of inference: axioms are theorems and if p and p implies q are theorems, then q is a theorem.

Euclid’s plane geometry provides an example of such a formal theory. The prepredicates are is a point, is a line and is on. One of the axioms might be:
(∀x ∀y x is point and y is a point and not(x = y)) implies ((∃z z is a line and x is on z and y is on z) and (∀w w is a line and x is on w and y is on w implies that w = z))

Contrasting this with “Through two points there is exactly one line.” makes clear that one would not choose to do ones logical thinking or expository writing in this formal language. The great triumph is it precisely sets forth what logical reasoning consists in and provides the foundations for a variety of major intellectual developments of the twentieth century. The logical reasoning in Euclid’s Elements is closer to such a theory than to Aristotle’s logic and with rare exceptions twentieth century mathematics proofs conform to the standard set by the logic systems as above.

In setting up a complicated formal theory the axioms chosen for the formal theory may be inconsistent, that is, there is a sentence p in the formal theory such that and both p and not p are theorems. It turns out that if this occurs, every sentence in the formal theory is a theorem: for any sentence q, (p and not p) implies q is a tautology and hence a theorem. Therefore if p and not p is a theorem q is a theorem for any q; you can prove anything. It may be very difficult to verify that a theory is consistent; examples abound in which inconsistencies are not at all apparent (One is tempted to wonder if this has some relevance in legal arguments and in religion.) In addition, in some cases, proving consistency is impossible. Various attempts, with little success, have been made to refashion logic so as to limit the consequences of contradictions

In chapter VI we describe how a formal language can be associated to a body of knowledge, including axioms capturing its truths and names the ways of interacting with the world which when taken together, provide the main part
of what will be called a Model of the body of knowledge. The logical reasoning in the body of knowledge can take place in the formal language and ways of interacting with the world are named by its predicates. A Model provides a precisely defined object with very little philosophical baggage capturing a significant part of its body of knowledge and enables clear comparisons and manipulations of bodies of knowledge.
Chapter V
Units of Knowledge

We take a broad view of what constitutes the world and knowledge of the world. Along the lines of “The Great Chain of Being”, the World may include God, gods, Saints, spirits, souls of the dead, souls of the living, human communities and nations, people, animals including the biology part of people, plants, the non-living physical world.

In thinking about the nature of truth and knowledge the tendency is to think of big things: our knowledge of the motions of the earth, the planets and the sun, the origin of the universe, the content of the Christian Bible, Darwin’s theory of evolution. Curiously in epistemology, even in philosophy of science, references to concrete bits of knowledge are rare. Although conceptually significant and much emphasized in history and philosophy of science, the motion of the sun, planets and stars make a modest contribution to the rise of our technical society, the more important scientific areas being the various branches of physics, chemistry and biology. All involve a variety of complex machinery and techniques for “seeing” the world. Similarly, knowledge pertaining to person-to-person relations, social organization, business management and government involve a variety of complex procedures, for example, legal procedures involved in governing human behavior.

In our history of the West’s notions of knowledge very little attention was given to what might be called practical knowledge, ranging from the techniques of the hunter-gatherers in providing the basics of life, to modern agriculture, medicine, private and public construction, and
warfare. In modern times practical knowledge and science have largely been combined in what we called the Thales Form. In the non-science, serious knowledge area, the Platonic Form is alive and well.

From a more pedestrian perspective, probably most of a person’s knowledge, including the most important parts, is acquired by age fourteen, that is language, how people socially interact and the rules of behavior. Probably of next importance would be knowledge of the basics of food clothing and shelter and the technology and organization of their production. Then there are more formalized rules of behavior and the topics taught in secondary school, schools teaching religion and the subjects taught and researched in universities and professional and theological schools. In addition there is the knowledge involved in producing a vast variety goods and services.

We stress the importance of taking account of ways of making contact with the world and take a broad view of what constitutes the world, including both the “physical world”, the “spiritual world”, man’s creations including his records of the past, his vast literature and his many and varied social structures. We take knowledge to be divisible into units, large and small, for example, quantum mechanics, how to ride a bicycle, astrology, commentary on the Gospel of St Mark, analysis of T. S. Eliot’s Hollow Men, psychology, pulleys and levers, patent law, setting broken bones, Feng Shui …

So far we have emphasized logical reasoning and contact with the world. We expand these ideas by defining a unit of knowledge to consist a combination of three interrelated objects, a text in the form a handbook or literature about which logical reasoning may be possible, a technology and a community. The technology consists of
Procedures, ways of interacting with the world, which, when executed, produces a statement of the outcome that can be recorded in the literature. The community is a group of people who know the unit, subscribe to its efficacy and directly or indirectly use it. The community includes a group of experts who are expert in using its text and technology, some of who refine them and expand them or who teach them. An expert or group of experts, acting as a team, execute Procedures producing recorded statements about the world, statements such as:

“James Smith is found guilty of murder.”
“These are pumpkin seeds.”
“The lead content in this sample of water $6 \times 10^{-9}$ is grams per liter.”
“God created the world.”
“The distance between the earth and the moon is 24,000 miles.”
“James Smith has one wife.”
“Monet painted in a painterly style”
“Jesus rose from the dead.”
“John Tyler was the tenth president of the United States”

We note for future reference that these sentences, if put in bold face in a formal language as described in chapter IV, are made up of proper nouns, pronouns and basic predicates, no “for all”s or “for some”s in contrast with general statements such as “Copper is a metal.”(For all x, x is copper implies x is a metal) Using a unit of knowledge will typically involve records of procedure-generated results and possibly appropriate literature, historical records and sacred texts. This material is part of the world which a unit can access using its Procedures.
Chapter VI
Models of Units of Knowledge

Recall, a unit of knowledge has a text, technology made up of Procedures, experts, and a community. The Procedures are the unit’s ways of interacting with the world and the experts are skilled in executing these Procedures producing accounts of the outcome added to text. The Community is the group of people who largely subscribe to the unit’s truths as embodied in the text. This is in contrast with the notion that knowledge can, in the main, be embodied in a text alone or in the mind alone. Typically the Text is of modest value in training new experts.

In using a unit of knowledge experts execute Procedures producing sentences in the text that are taken to be true sentences. Then the experts, using logic, decide what to make of these true sentences in relation to the motivating aims of the undertaking. If an expert measures the temperature in your office and finds it to be 72 degrees, he is reasonably confident that “The temperature in the office is 72 degrees.” is true. If the text for office comfort, possibly based on studies, specifies that temperatures between 70 and 73 degrees are comfortable, then the expert deduces that your office is comfortable with respect to temperature.

Many units of knowledge involve considerably more than logical reasoning, Procedures, experts, and a community, especially in the case of social knowledge: morality, religion, law, politics, etc. We explore how far the combination of these four interrelated elements captures a plausible account of truth in bodies of knowledge. They also
provide tools for unraveling various “knowledge” questions. To capture the logical reasoning part of a unit of knowledge we associate to it a formal theory, that is, a formal language plus axioms and hence theorems, which we call a *formal theory for the unit of knowledge*. This theory is as follow:

The prepredicates, pronouns and proper nouns of the formal language are bold face renderings of prepredicates, predicates, pronouns and proper nouns in the unit of knowledge’s text, chosen so that (ideally) any sentence in the text can be reformulated, without changing its meaning, into a sentence obtained by removing boldface from a sentence in the formal language; roughly, the text can be translated into the formal language and in reverse. Furthermore the axioms of the formal theory are chosen so that the sentences in the text viewed as being true, correspond to theorems in the formal theory. There may be a very large number of axioms as, for example, a large number sentences from the Bible. Efforts should be made to insure that the axioms chosen should be consistent, that is, they should not lead to contradictions. Furthermore, arrange the Procedures of the unit of knowledge so that when a procedure is executed it produces a sentence in the theory, which, when bold face is removed, says the outcome of the execution. Note Procedures, when executed, produce a sentence made up of basic predicates, proper nouns and logic words excluding ∀ and ∃. We take the basic predicate to be the name of the procedure and, when executed, produces the procedures name with its variables replaced by proper nouns and possibly preceded by not. The name of the procedure that produces *Mary loves John* is *x loves y* and *Mary and John* are proper nouns. Executing *x loves y* may produce *Mary loves John* or *not Mary loves John*.
We define a *Model* of a unit of knowledge to be the formal theory for the unit, as above, together with the unit’s procedures rendered in boldface, experts and community. Thus a Model consists of four interrelated parts, formal theory, procedures named by basic predicates in the formal theory, a community and experts in the community who execute procedures producing sentences in the formal theory.

We emphasize that the theory is related to its unit’s procedures by their being named by basic predicates (prepredicates such as *is a man* and *is happy* and *loves*, combined with $x$, $y$, … and all logic symbols except $\exists$ and $\forall$) in the formal text. The possible outcomes of a procedure, as in the example $x$ *loves* $y$ above, when it is executed, produces a sentences in the model with its variables replaced by proper nouns and possibly proceeded by *not*. For example, the procedure named *$x$ is guilty of robbery*, when executed might produce *not George Smith is guilty of robbery*, when George Smith is found not guilty of robbery.

In our definition of a model we add as part of it, its history which at any point in time is a document giving, in the language of its formal text, theorems which have been proved and their proofs and a collection of outcomes of the executions of its procedures judicially chosen and added as axioms producing additional theorems and proofs; thus a model is a dynamic object. For convenience we have specified that our formal text include a fixed logic part as described in chapter IV.

Land surveying provides a familiar example of a Model. A Model named “Land-Surveying” could have its formal text a formalization of Euclidean geometry by making all of the text of a modern mathematical account of Euclid’s Elements in bold face and axioms as in the text. For its
procedures take the collection of procedures surveyors use, i.e., measuring distance with a long tape measure and large rulers and measuring vertical and horizontal angles with a transit; its experts are certified surveyors and its community, the general public, much like the ancient Egyptians.

All of our ordinary knowledge about the material and natural world of a more or less technical sort, how to make beer, build houses, grow crops... have straightforward mature Model formulations. (In Chapter X “mature” is described). More or less, for the formal text, put a handbook for the subject into bold face and set down the procedures, which are described in the handbook, make “principles” axioms and specify the community and the experts.

A Model that produces That chair is green might be formulated in a way that includes that amazingly complex measuring device which takes months to learn how to operate, namely, our vision apparatus.

To admit of having a Model is not a very stringent requirement on anything passing for knowledge, especially if we allow the absence of non-logic axioms and/or procedures so as to include pure mathematics and riding a bicycle.

The procedure x is an apple, when executed, has possible outcomes such as this is an apple and not this is an apple, outcomes which make an important semantic contribution to the meaning of “apple”. A store of data of these outcomes including a variety of occurrences of this is an apple leads to a reasonably well-defined concept in a community and ∃x x is an apple is a mildly interesting axiom or theorem. The possession of the concept “apple” as rendered as a procedure is a significant bit of knowledge and a good deal of knowledge is of this type. In contrast, the theorem that the sum of the angles of a triangle is 180° in
the Land-Surveying Model yields a much more powerful example. In the informal unit of knowledge’s text, instead of its Model language, suppose A, B and C denote markers on three hill tops and angles are denoted by ABC for the angle in degrees given by the lines AB and BC. Then, ABC+BCA+CAB=180° is a theorem in Euclidean geometry. Suppose executions of the angle measuring procedure yield ABC=26° and BCA=70°. Then the associated model says that of the possible values yielded by measuring BAC, between 0° and 180°, the one that will be produced is 180°—26°—70° = 84°, saying something considerably stronger about the world than is a volcano yields. This example exhibits the power of prediction. Although we will very much elaborate predictive power, this example actually goes a long way toward accounting for the vast expansion of science and technology.

Of course some units of knowledge are about the world in the sense of using carefully defined names of things in the world, which can be reliably applied, but fail to have predictive procedures, for example, botanical classifications. Astrology is a good example of a unit of knowledge with no predictive power. It has an elaborate formal model involving the positions of the heavenly bodies and human events along with procedures yielding such sentences as “Mary Smith was born at 6:37 AM, July 19, 1964.”, “Susan Jones will receive some good news this week”. The prediction is understood to give a tendency for a broadly described event to occur with the consequence that no possible outcome contradicts the model (p contradicts the model if not p is a theorem.) Given a person’s birth date and the present date, astrology might predict that the next week will be a good time to form a new friendship. No turn of events could
contradict this in as much as there would always be plausible accounts for any outcome.

Many Models of an applied sort have predictions based on plausibility rather predictive procedures. Like astrology, they have no outcomes of procedures that contradict the theory. This occurs in medicine: on the basis of patient reported symptoms and medical tests, the model provides a diagnosis, a treatment and predicted outcomes, which include the possibility that diagnosis was incorrect or the treatment failed in this instance. A history of success an failures may recommend its use and the recent emphasis on statistical analysis of treatment outcomes, including simple accounts of success rates, is increasing medical Models with predictive power.

In keeping with our classical roots, most people including scientists are vaguely Platonic in their outlook. The scientists view themselves as discovering laws or approximate laws of nature, which are true statements about what is out there in the world. But the power of accurate predictions is paramount.

Fussing about with what it means to say that Maxwell’s equations are true is of little concern aside from their predictive power for such phenomena as alternating electric current. Largely, the scientists view is that there should be no difficulty in developing an adequate theory of truth that accommodates the primacy of experimental evidence. Tarski’s semantic definition of “true”, “Today is Wednesday.” is true if and only if today is Wednesday ([10]), would seem to be quite adequate. This contrasts with the Platonic Form where the nature of truth is a major concern holding out the possibility of important statements about the world, which are, “truths of complete certainty”. Astronomy, the age old largely religious subject of modest
practicality, gradually became more scientific; now rechristened cosmology, with its black holes, dark matter and expanding universe, might be viewed as highly speculative, but as observed in a review of *The Extravagant Universe*, “cosmology is now a true observational, experimental science, securely grounded in the messy practical realities of making measurements.”

Although in our Model formulation, defining ideas, logical reasoning and procedures are not novel, the emphasis and precision differs very substantially from the traditional representation theories of knowledge (sentences correspond to states of the world). Traditionally, a well-established body of knowledge about the world consists of a theory analogous to our formal model idea, that is, a collection of concepts, a set of first principles specifying relations between the concepts and a collection of logical deductions from these principles which accurately describe certain features of the world, Euclidean Geometry being the gold standard. The problem with this approach is finding an account of how a theory is related to the world. To explicate this connection, the tendency has been to focus on some version of deep thinking (a’ priori), on direct experience, on consciousness or on the truth of statements such as “The needle on the meter is at 6.5.” The theory is the principal part of a body of knowledge and the task of epistemology is to identify how a theory is connected to the world and specify what makes a sentence true.

Our most significant departure from traditional epistemology is that a sentence only makes sense in the context of the Model containing it and in our replacing “knowing” by “aggressive interaction”. Traditionally, knowing something about the world is viewed as a passive activity, that is, what is out there that we know is
independent of our knowing. In contrast, Models are very much human creations. In isolation a formal language is just a manipulation of symbols game and most procedures without some theoretical context are meaningless (The point made in Quine’s rejection of the “analytic-synthetic distinction”.) Our conception of knowledge is more analogous to the feature of quantum mechanics that dictates that to make sense of the goings on in an isolated system, the observer must be included in the system. Our “Model” concept provides a simple solution to the problem of relating theory to the world but we are then left with sorting out in what sense knowledge can be objective. We address this in XVI.
Our procedures-experts combination in a Model specifies the existence of a collection of experts with a teachable expertise who can with some success identify situations where the unit of knowledge is applicable and in these situations execute its procedures, producing as their outcomes sentences in the formal text.

Recall, a procedure in a Model is named by a basic predicate in the formal model. A procedure such as The temperature of \( x = y^\circ \text{F} \), when executed, produces a sentence such as The temperature of the water in my flask is 190\(^\circ\) F. The execution of The temperature of \( x \) is \( y^\circ \text{F} \) has many possible outcomes, namely sentences obtained by replacing \( x \) by a proper noun and \( y \) by a number. Executing The temperature of \( x = y^\circ \text{F} \) (dipping a thermometer into the water) picks out one of these sentences. Recall in naming procedures we include combining names of procedures with and, or, implies and not; (x is water)and(the temperature of \( x = z^\circ \text{F} \)) names the procedure which the expert may elect to mean simultaneously measuring whether \( x \) is water and its temperature. Executing \( x \) is water would produce the content of the flask is water or not the content of the flask is water, which may be added to the axioms.

We view the execution of a procedure as a well defined interaction between the expert and the world whose outcome is a sentence such as The temperature of this water is 126\(^\circ\)F, That chair is blue, God made the universe in six days, There are three apples in the bowl, The distance between the earth and the moon is 240,000 miles, My
wife said I should mow the lawn, which make sense in the context of their models. Any law library will set forth a vast collection of legal procedures that would give procedures in our sense. Any science library will contain volumes titled “Standard Laboratory Procedures for...” which specify in detail how specific measurements are to be performed. Any paper which includes results of experiments, carefully spells out the procedures used. In matters involving religion, referring to a sacred text is a Procedure which when executed produces a quotation from a sacred text.

In general procedures and their intertwining with the formal model are much more complicated than our boiling water example. A great variety of complex procedures interrelated in a model are involved in community life in determining that a crime has occurred, finding, arresting, charging, trying and sentencing to prison, if found guilty. Similarly complex, in Milliken’s experiment to determine the ratio of the electric charge of an electron to its mass, the procedures consists in viewing, through a microscope, a small oil which has an extra electron or two on it and which is suspended in space by an electric field. One measures the diameter of the oil drop and the strength of the electric field needed to suspend the oil drop and via the theory calculates the ratio of charge to mass as -1.758 ×10^{-11} C/kg. In both cases elaborate interaction with the world is involved and only make sense in the context of some version of a model.

An important feature of most procedures is their universality in the sense that a Procedure of one community can be taught to another community incorporating it in their language and when executed each community will produce the same outcome, suitably translated, for example, The temperature of x is 98.6°F.
Exceptions to the universality of procedures does occur, for example, in situations of valuations, judgments as to the merits of art objects or the seriousness of a particular moral lapse. This non-universality can be eliminated by including in the procedure the person or community involved.
Chapter VIII
Simple Examples of Models

Following standard expository style in the sciences and mathematics we give simple examples of Models that are as simple as possible while including the main ideas and accommodating subsequent elaborations.

Recall a model includes procedures for interactions with the world which are named by basic predicates in the model’s formal theory. Some examples are:

- \( x \) is an American
- \( x \) is convicted of grand larceny
- \( x \) owns property \( y \)
- \( x \) is a Christian
- temperature of \( x \) is \( y \)

Typically the power of a unit of knowledge can be expressed in a Model by the Model’s non-logic axioms and theorems that include \( \forall \), that is, general principles. One can construct Models having only one non-logic axiom such as the following:

\[
\forall x \ (x \text{ is a American implies } (x \text{ is right-handed implies } (x \text{ holds the fork in his right hand when eating}))
\]

\[
\forall x \ (x \text{ a man and } x \text{ is convicted of grand larceny) implies } x \text{ spends at least two years in jail.}
\]

\[
\forall x \ (\text{The New York Times printed that } x \text{ was president of the United States implies } x \text{ was president of the United States}.
\]
∀x ∀y x owns property y if and only if x owns property y is recorded in the courthouse.

∀x x is a human implies x is a sinner and God in his infinite wisdom decides if x goes to Heaven or x goes to hell.

∀x x is a married woman implies x is older than 15 years.

∀y Hopper painted y implies the painter of y was largely concerned with colors and their combinations.

∀x x is water implies (x is boiling if and only if temperature x is 212°F)

The last example above is an axiom in a Model for the well known bit of knowledge from physics that water boils at 212°F, or more accurately, in its liquid state water, is boiling, if and only if its temperature is 212°F. We describe this example in considerable detail in Chapter IX and use it in discussing the works of Kuhn.
Let us begin by considering in detail the simple case of a student testing the boiling water model with one non-logic axiom:

∀x x is water implies (x is boiling if and only if temperature x is 212°F)

The student is given a flask, which he fills with water, an electric coil which he uses to heat the water and a thermometer to measure its temperature. The coil has a control to vary the amount of heat produced. He puts the heater in the water and turns it on. The water slowly heats until it boils, after which the student turns up the heat several times. During this process he takes ten temperature readings, noting each time whether the water is boiling or not. If he is reasonably careful, each time he measures the temperature when the water is not boiling his reading will be below 212°F and each time when the water is boiling, even though it boils more vigorously as the heating control is turned up, its temperature remains at 212°F. He generates ten pairs of measurements each consisting of a temperature and boiling or not boiling.

In preparing his laboratory report, the student might write something like the following. Let T denote the temperature of the water and W the water in his flask. The physical law applied in his case is

(1) T = 212°F if and only if W is boiling.

“Ten measurements, measurement pairs, temperature and not boiling or boiling, were taken and listed in the table
below and they all confirm the law stated above.” Finally he gives a table listing his measurements.

Converting the above to a Model, take the prepredicates for the formal text to be is water, is boiling and has temperature $^0$F giving basic predicates such as x is water, x is boiling and x has temperature $y^0$F. It would have one non-logic axiom:

(2) $\forall x \ x$ is water implies (x has temperature $212^0$F if and only if x is boiling)

To use this add a proper noun S for the students sample of water Then,

(3) (S is water) implies ((S has temperature $212^0$F) if and only if (S is boiling)) is a theorem deduced from (2) and the axiom in logic that says if something is true for all x then in is true for any particular thing, in our case, S. In as much as the formal model’s mnemonics are in English we could take the class instructor as the expert and for the community all English speaking elementary school graduates. Name the model “BoilingWater”. It should be thought of as a dynamic bit of knowledge in that sentences stating measurement outcomes can be added as axioms and in expanded form procedures would be added to access records of past measurements viewed as part of the world. If the student measures the temperature of his flask of water and produces S is water and S has temperature $126^0$F he adds it as an axiom and could then construct a proof that not(S is boiling) is a theorem and a prediction of what he would see of the state of the water, that is, that S is not boiling.

At a practical level, our boiling water model would be used by an engineer designing a water heater. For example, if his design is to include preventing a hot water heater bursting from the water boiling, the engineer’s design might
include a valve that opens when the temperature of the water reaches 200°F.

A Model is a guide to action that sometimes works. The model BoilingWater we have been discussing is well established in the sense that there are a vast number of recognized situations where it works. (Of course at some point man discovered that it does not work on the top of a mountain and then corrected it.) Confronted with a reasonably familiar situation, a trained person makes a judgment that the Model is applicable and he is usually right. A successful Model has several situations where it has given correct results and an expert can judge with some confidence when it will work (For example if it is sea water it will not.)

There are a tremendous number of Model’s similar to our BoilingWater example in the experimental sciences and their associated engineering practices, that is, in the Thales Form. Hundreds of such Model’s go into the design and manufacture of an automobile and produce a vehicle which will, in most cases, operate as predicted for more than ten thousand miles. Groups of small Model’s like our example are organized into very large ones. For example, consider the contents of a freshman physics text. Typically it has about fifty sections divided among mechanics, heat, sound, light, and electricity and magnetism, each section giving a small Model and all of them linked together by overriding concepts of distance, mass, and time, and organizing principles such as that the units of all variables are expressible in terms of the units for the fundamental variables: distance, mass and time.

Although our units of knowledge involve formal languages, in practice of course, reasoning and its interactions with procedures are done on an informal level.
Typically the applicable situations are described in the language of the model and possibly other models (What does “water” mean in our example?). We claim that the notion of Model captures scientists’ basic conception of scientific knowledge. The informal versions of a Model’s parts provides a framework, a vantage point, from which to view some aspect of the world or some other informalized Model. When in the Thales Form, theoreticians work on the theory of the Model, discovering and more or less formulating conjectures and sometimes proving interesting new theorems, often aimed at accounting for inexplicable experimental results. Experimentalists execute procedures in new situations and devise and execute new procedures to resolve questions concerning predictions of the Model. Engineers in turn convert these Models into useful applications. Without the context of a Model, a procedure is meaningless as is theory without its associated procedures.

We emphasize that in our Model construction, procedures should be given at least as much weight as the theory. Although most scientists take a somewhat Platonic view of the world claiming that they discover the “laws of nature” or approximations to the “laws of nature”, when it comes to serious questions of truth, in the hard sciences, experimental evidence takes precedence over all other considerations (a claim disputed by Kuhn and postmodernists, see Chapter XVIII). It should also be noted that most of modern technology and engineering is the result of generalizing and refining the experiments of science. For example, in the middle of the nineteenth century, experimental physicists, trying to isolate the flow of electricity, developed vacuum tubes through which they were able to pass a current. This apparatus was generalized to create the oscilloscope in the 1920’s and the Television
tube in the 1940’s. Or witness recent developments in medicine.

It follows from General Relativity theory that two points do not determine a straight line, there are experiments falsifying this. Where does that leave building a house? Are Euclid’s straight lines still out there for the builder? The answer is yes. He uses a Model in which this is true. Ask a carpenter. Working with his Model, he would say, "Of course, look at that straight portion of the picket fence I’ve just put up with pointed, main posts every ten feet and the string I’ve stretched from the point of the first post to the point of the last. Note that the points of the other posts, except the third one, just touch the string. They are on the straight line from the point of the first post to the last except the third one isn't." That is, there are straight lines out there if there are apples out there. Are there unicorns out there? We know the Model, we just do not have any experiments that have produced "This is a unicorn."

Probability provides an interesting Model example. I take a quarter out of my pocket and ask what does it mean to say that when I flip it, the probability of a head coming up is 1/2 (or between 1/4 and 3/4)? A considerable thought and energy have gone into trying to reduce such sentences to sentences not involving the word probability or other probability terminology. For example, in probability talk, if you have an urn containing twelve black balls and six white balls and you pick out a ball at random, the probability of choosing a white ball is 1/3. Now all you have to do is explain what "random" means. The content of a standard text in mathematical probability, together with experts in applying it, provide a good example of a bit of knowledge in the sense of our Model construction. Such texts devote some prose to get across the idea of probability by examples and
such phrases as the probability of a head coming up is 1/2 means that if you flip a coin a large number of times, very close to 1/2 the times, a head will come up. But the substance of the book is devoted to such things as, if the probability of a head is 1/2 and you flip the coin twice, the probability of getting two heads is 1/4. In other wards, no precise meaning is given to “probability”; it occurs in a Model which has proven very successful. In Las Vegas very little attention is given to reducing "The chance of throwing a seven in craps is 1/6." to a non-probability statement even though very important matters depend on this calculation. The associated Model would formalize: Probability theory plus “These dice are carefully manufactured and tested to be fair and hence the probability of the two dice being thrown coming up with seven is 1/6.”
Chapter X
Models in Mathematics

Probably the most universal and useful unit of mathematical knowledge is the natural numbers, the counting numbers, 1, 2, 3, ... . They are a basic part of mathematics and a basic part of epistemology, especially in idealism philosophies. The dominant branch of epistemology asserts that there are statements about the world that are unequivocally true and whose truth does not depend on experience of the world, for example, Kant's a' priori statements. The existence of such sentences is viewed as obvious, 2+3 = 5 being an important example. Thus in philosophy it may be important that "2", "3" and "5" are names of things in the world and in addition satisfy 2+3=5. Russell and Whitehead's Principia, in a way, gives things that can reasonably be named "2", "3", and "5" and "2+3 = 5" becomes a theorem, but their setup has axioms whose a' priori truth must then be addressed.

Although some mathematicians are mildly interested in whether they discover or invent new mathematics, almost everyone agrees that an axiomatic approach to the natural numbers is all the foundation one needs for them. Any mathematician can give several models for the natural numbers, which have as theorems everything we know about numbers including 2 + 3 = 5. But what about counting things in the real world? Aren't numbers names of some things in the world, isn't 2 + 3 = 5 a statement about counting in the world? Apple harvesters, George and Bill, come in with baskets of apples which they empty into a central collection basket. The foreman uses the simple model consisting of the addition tables he learned in school. He adds to his model B, G and T for the number of Bill's,
George's and total apples and the axiom \( B + G = T \). He asks Bill and George how many apples they brought in and they respond 2 and 3. He has the theorem \((A = 2 \text{ and } B = 2) \implies (T=5)\). And hence his model predicts \( T = 5 \). Just to be sure, the foreman counts the number of apples in the collection basket. Suppose he finds that there are six. This may reflect a serious flaw in our theory of counting or a major shift in the structure of the universe. Possibly, but a more likely explanation is someone made a mistake. There are many situations where our counting theory works well and many where it doesn't, for example measuring a volume of cocoa by counting rounded teaspoons of cocoa. When there is a failure we blame the situation not the theorems. Possibly this is the defining characteristic of a fully mature unit of knowledge. In the nineteenth century Newtonian mechanics was probably viewed as close to a mature subject, possibly with the undeniably mature aspect sorting itself out as the calculus.

Pure mathematics has Models but no procedures. Rather, it has sort of pseudo-procedures, procedures involving interactions with other models. For example Number theory has as its subject matter the natural numbers, 1, 2, 3, …, which in turn are given by a model. The model usually used is an axiomatic formulation of the natural numbers devised by Giuseppe Peano around 1890.

Until early in the nineteenth century, mathematics was viewed as about the world much as the way physics is today. The natural numbers were viewed as objects in the world, a Platonic viewpoint, and a great deal of effort, in philosophy, went into trying to make sense of this.

With respect to what we now call the real numbers, expressed in possibly infinite decimal form, is quite a new idea. Initially real numbers were viewed as measurement
outcomes and did not get sorted out until the 1850’s, starting from a postulate approach for the natural numbers. Mathematical papers, leaving aside discussion, can be straightforwardly translated into a formal theory.

Present mathematics research has taken a somewhat structuralist tack. The undefined terms gain their meaning from their place in the model as a whole. In Euclidean Geometry, lines are those things determined by two points; points are where two lines cross. A natural number gains its meaning by being one of the natural numbers making up a system which in turn has various properties by virtue of being in larger systems. Instead of pondering the meaning of “a number”, concentrate on setting down a set of relations between numbers which largely characterizes them.

The extent to which a subject is understood and worked out depends of course on success in proving fundamental theorems, but developing conceptual frameworks is one of modern mathematics’ principal functions.
Chapter XI
Models for Social Knowledge

Based on the perspective of a community, the world can be divided into the physical world, the social world and possibly the spiritual world. The social world of a community consists of portions of its members’ minds, content of its texts, the artistic aspects of works of art, and such. Depending on the community, it may or may not have a spiritual world, which is part of the social world, part of the physical world, spread over both, or neither. More specifically, we view social knowledge as knowledge of social customs, rules of behavior, legal systems, systems of government, history, art, religion and the humanities in general.

Our Model approach to units of knowledge may be straightforwardly applied to both knowledge of the physical and the social worlds. For example, the text of a Model could have a theorem formalizing, “A man may lawfully have only one wife at a time.” The community might be the population of the State Minnesota and the experts might be the states law enforcement people. The sentence could be an axiom, possibly from a sacred text or deduced from other more basic axioms. Actually most Models involving the social world involve both social and physical parts of the world. An account of the activities of the Greek gods is probably an example of a pure social body of knowledge while automobile speed limits involve a combination of the social and physical worlds. In our terminology, one can associate to a unit of knowledge a Model, that is, a community, text, procedures and experts. Typically, in a Model for a social unit of knowledge the experts are
lawyers, judges, theologians, historians, art critics … . Axioms are chosen from religions, constitutions, legal documents, historical records, etc.

The various types of procedures: test and verified, and Models: verified and empirically valid, described in Chapter XIV, also apply to these Models. Some examples of social procedures are:

- **x is the governor of Texas**
- **x is y's bank account balance on day z**

Some social sentences are:

- \( (x) \) x is a impressionist painter implies x is Claude Monet.
- **Mary is George's wife**
- \( (x) x \) is a man implies x should not have more than one wife at a time

Some slightly more complicated examples are:

- \( (x) \) x is eating implies x holds his fork in his right hand
  - which is a physical world sentence.
  - The fork idea might be rendered as social knowledge by,
  - \( (x) \) while x is eating, x should hold his fork in his right hand
  - which might be accompanied by a physical world statement,
  - \( ((x) \) while eating x holds his fork in his left hand implies x will be viewed as uncouth) and \( ((y) y \) is viewed as uncouth implies (y is rarely invited to dinner))

While the physical world is mostly a mystery to humans, at any point in time, anything in the social world of a community is accessible to some expert. For example, the ninth amendment of the U S Constitution. On the other hand, what Herbert Hoover was doing at 10 AM June 15, 1925 is probably not in the present social world. Of course
there may be a record of his activities available now that is accessible and hence part of the social world now.

The social world of the community can be divided into various categories, for example,

- In the mind of a member
- In all of the minds of the community members
- Regulating documents, such as laws
- Religious texts
- Legal texts
- Art objects
- Accounts of the past
- Accounts of other communities

Many communities welcome and encourage expansion of knowledge of the physical world, a world that is largely unchanging. In contrast, the social world is largely viewed as timeless, with things that are obviously changing, being segregated into “style” or “fashion”. In fact the social world is often drastically changing, sometimes due to the expansion of physical knowledge. The basis for morality may be in the physical world, but over time such rules may be drastically revised, for example, as the number of children in families.

Although our notion of a model of a unit of knowledge applies equally well to knowledge of the physical and social worlds, there is a huge difference in how the two worlds are related to time. Except for minor human tinkering or from the aspect of humans as part of the physical world, the physical world is timeless. Humans have learned a great deal about the physical world and with this knowledge have significantly modified the world, but in the big picture, the changes are modest. In contrast, at least from 10,000 years ago to the present, the social world has steadily and drastically changed. Although some communities claim that
a substantial part of the social world is timeless, such claims are historically difficult to substantiate. Aside from such issues, the social world often drastically changes. For example, with respect to most US communities, over a forty year period, the requirement that a woman should be married to the man who sires her children, to now, when a woman may socially acceptably have a child without being married and may be the only adult in her family. A significant part of the social world is devoted to regulating social change. In contrast with knowledge of the physical world, anchored in the timeless physical world, which is independent of humans, a community’s social knowledge is only anchored by its physical underpinnings and it’s changing social world having remarkably small inertia.

Models associated with a fixed community may have a variety of expert groups and procedures. Models associated with folkways, customary ways and ordinary conventions, will have all but young children as experts. The experts will judge appropriate and inappropriate behavior, typically using rules involving “should”, “ought” and “must”. For civil law the experts will be lawyers and court officials. For serious deviant behavior, crimes, the experts will be the police, criminal lawyers and court officials.

Recent research results have been interpreted as meaning that some moral principals are genetically specified and in consequence basic moral rules are universal, in our Model formulation, community independent. A response to this has been granting the accuracy of the research but responding that it is still up to individuals and communities to decide what rules to use, that is, to go with or against “nature”. For example, although a man and women joining to form a family and producing children is universal, one community my allow any pair of persons to join in matrimony while an
another may entirely prohibit it. Most communities settle on some ought and shall intermediary. The Model including a version of the above probably would have “ought” statements as theorems and theorems relating ought and procedure outcomes attesting to the community’s conformity with the “ought” statements.

One society studying another will commonly read into it its own social framework. This is most notable in early anthropology, for example, in “The Golden Bough”. Modern anthropologists despair of defining “religion”; although features of western religions are shared by other cultures, perhaps religion, as a concept, is a western invention. Our division of the world into physical, social and spiritual is vague and probably a Western concept.

From our model perspective, Religions such as Christianity, when viewed as a body of knowledge, may have a large collection of axioms, namely, the sacred texts. Also they involve the language and events of everyday affairs as well as states of mind, and in that sense, are rich in procedures. But they are often far from being constrained by the procedures.

One approach to ethics and political theory is to set forth basic social axioms for a formal theory in our sense and deduce resolutions of current political and moral disputes. An early modern example of this is Kant’s a’ priory categorical imperatives, for morality, a somewhat pedantic formulation of “Do onto others as you would have them do on to you.”, namely, “So act as if your maxims should serve at the same time as the universal law (of all rational beings)”. This approach of basic statements continues at present in Isaiah Berlin’s account of equality and liberty, John Rawls’ political theory and Ronald Dworkin’s axioms of life’s intrinsic value and personal responsibility. Certainly
these works broaden one’s thinking about present morality and politics but being a theorem in these settings probably has limited force. They provide an interesting contrast between knowledge of the physical and social worlds. The laws of physics are specified by the physical world; it specifies that $E = mc^2$. In contrast Kant and similar thinkers wish to modify the social world. They attempt, by deep thinking, to provide ways the social world should behave.

We take issue with Rorty’s "account of inquiry which recognizes sociological, but not epistemological, differences between such disciplinary matrices as theoretical physics and literary criticism" ([6]). We need to clarify what we mean by these disciplines. One might conjecture that theoretical physics consists in proving theorems in the model part of a physics Model. Although some of this goes on, mostly what theoretical physicist do is manipulate a Model by modifying and adding non-logic axioms, guided by analogy, deduction, calculations and physical intuition. They justify their results by their success in producing a model which explains known experimental results (converts experiments into verified procedures) and suggests new test procedures. Thus theoretical physics involves the procedures of Models in an essential way and lays great stress on its procedures being reproducible. In contrast with theoretical physics, most schools of literary criticism explicate works of literature from models such as Marxism, psychoanalysis, archetypal criticism, structuralism, etc. without much in the way of test procedures. In addition, the execution of a procedure, that is, an analysis of a work of literature, is not meant to be reproducible, its outcome being its enlightening and influencing people in the field and in broader segments of society.
In Chapter XIX, “Platonic and Thales Forms of Knowledge”, we elaborate the differences between social and non-social knowledge.
Ch
[244x38]apter XII
[266x659]Empirical Truth

In many situations what one wants in a Model is that it reliably makes accurate predictions about the world. Experts are able to judge situations where it can confidently be applied. If the execution of a procedure of the Model Boiling Water yields

\((S \text{ is water}) \land (\text{temperature of } S \text{ is } 212^\circ F)\)

and we put this in as an axiom, then, by virtue of \textbf{A is boiling} becoming a theorem, one can confidently conclude that \textbf{A is boiling}. Of course another possible outcome is \textbf{not A is boiling}, contradicting the model, requiring an explanation which may or may not be easily explained. This is the nature of real life predictions. The “empirical truth” is never certain. It should be pointed out that most sentences in a Model cannot be the outcome of an experiment or any collection of experiments. \textbf{Water boils at } 212^\circ \text{F}, \text{that is, } \forall x (x \text{ is water}) \implies (x \text{ is boiling if and only if temperature of } x \text{ is } 212^\circ \text{F}), \text{is not a sentence that can be produced by any procedure. Procedures can only produce collections of sentences consistent with this sentence. Just because the sun has risen every day in the past, need it rise tomorrow? To address this we have an elaborate theory including the motion of the planets around the sun or of Helios riding his chariot across the sky each day. In each case, "The sun rises every day." is a theorem in the text of a unit of knowledge which has survived a variety of tests. One might ask, just because the unit of knowledge has worked well in the past, is that any reason to think it will do so now? Other than references to other Model’s set up to discuss this, for example, Model’s for history, of course not. The point is
we use Model’s with considerable success and in many particular situations, use them with confidence and hope for the best. In addition, the experts may be prepared to investigate and explain particular outcome failures and possibly modify a Model. For the researcher in the sciences, a failure is possibly a great opportunity and at any time there can be major unresolved questions.

In general, a Model should not be viewed as fixed. A sort of infinite regression phenomenon occurs if one tries to make a Model completely precise. For example, in critically thinking about our boiling water model, perhaps with a view to applying it to a new situation, one might have to think about “water”, which in turn would require its own model. In other words, in critically examining some aspect of a Model, one constructs another Model. It is like the question, what holds up the earth. It sets on the back of a large turtle, and what holds up the large turtle, it sets on another large turtle and it is large turtles all the way down. Similarly, its Model’s all the way down.

The Thales Form of knowledge is related to Popper’s “falsification” definition of empirical validity in science. An enduring problem for the epistemology of science has been the problematic nature of inductive reasoning. From the sun’s having risen every day can we conclude the “law”, “The sun rises every day.”. One would seem to need this if one wants the laws of science to be true statements about the world. Popper [5] proposed that the substitute for induction should be failure to falsify, that is, in our terms, failure to find a procedure whose outcomes contradicts the Model. Failure of persistent efforts to find experimental evidence contradicting a unit of knowledge is to provide definitive grounds for it’s being true, a very Platonic perspective. The “falsification” idea has been taken seriously in Philosophy
of Science and the humanities but ignored by scientist. A theory’s predictions being mostly false is no reason to reject it if in some known situations it consistently gives correct predictions; the accent is on power.
Chapter XIII
Formal Truth

Most theories of knowledge are preoccupied with what it means for a sentence (proposition) to be true. For our Model approach, being a theorem and being the outcome of a procedure captures the relevant features of "truth". Nevertheless being able to talk about true sentences is very convenient. Logic becomes extremely cumbersome, if one cannot say things like, if \( P \) is true and \( P \) implies \( Q \) is true, then \( Q \) is true, or in real life, "If it is true that the interest rate is 5%, this bank balance can't be right." For our purposes a simplified version of Tarski's semantic definition of truth ([8]) provides a simple solution: "Today is Monday." is true if and only if today is Monday. One can add symbols true, " and " into a formal model together with appropriate grammar and axioms governing its use so that the theorems become true. That is, if \( p \) is a sentence in a theory, specify that "\( p \)" is a noun and that \( \text{true} \) if \( p \) is an axiom. Probably we should think formally true to accommodate the Gödel incompleteness phenomena. As we specified, the outcome of a procedure is added as an axiom, hence a theorem and hence true and becomes history.

If you ask an expert who has just measured the temperature of water in a flask, "Is it true that the temperature of the water is 130ºF?", and he answers yes, it probably means he has applied a standard procedure and gotten an answer of 130ºF. He would admit that there is remote possibility that he made a mistake. But if he planned to use the model to make predictions, it would be reasonable for him to describe the outcome of the execution of a procedure as true. Then the predictions following from the
execution of a procedure become true with the same reservations.
Chapter XIV
Types of Knowledge

Units of knowledge, can be classified using an associated Model as follows. First, on whether the Model has any procedures and then whether it has any non-logic axioms. If it has procedures but no non-logic axioms we call it a classifying Model; for example classifying songbirds. If it has non-logic axioms, it then has theorems and one can have procedures whose outcomes are either theorems or contradict theorems. For example **x is water and x has temperature 212°F implies x is boiling.** We call such a procedure a test procedure and we call it a verified procedure when it has been thoroughly explored and reliably produces a theorem when executed. If a Model has one or more verified procedures we call it verified and empirically valid if all its test procedures have been verified. To what extent do these verified procedures constrain the model? One could have a Model with most of its theorems unrelated to its verified procedures. We define a Model to be verified and empirically valid if all its test procedures have been verified if each of its non-logic axioms is contradicted by the negation of one of the test outcomes. Finally, we consider the reproducibility of procedures.

The procedure, **The tape measure of length of my desk is x,** is highly reproducible in the sense that if done several times by various people and at various times they all get the same answer. **The picture x is done in a painterly style,** is less reproducible. We define a Model to be empirically complete if it has non-logic axioms, it is constrained by its verified procedures, its procedures are reproducible and vigorous efforts to find and execute test procedures which
contradict its model have failed (This is of course time
dependent and more comprehensive than Popper’s not
falsified.) A major feature of hard science is its extreme
skepticism. Vigorous efforts are made to find procedures
whose outcomes may contradict a theorem. This stands in
marked contrast with anecdotal evidence, simply picking out
cases where events turn out to be as predicted by the theory
and putting them forth as supporting the theory. Most of
well established parts of hard science consist of large units
of knowledge which are empirically complete. For the most
part, everyday knowledge about the physical world is
similar to the hard sciences in this regard. The production
and preparation of food, clothing and shelter all fit our
Model setup having outcomes such as **This is a pumpkin
seed** and theorems such as **If a pumpkin seed is planted in
the ground, then the following sequence of events will
occur:**…. Law, government, social norms and folkways
involve a great variety of units of knowledge with
empirically complete Models.

Many Models of an applied sort have predictions based
on plausibility but no test procedures. Like astrology,
instead of test procedures, they have procedures all of whose
outcomes are theorems (The execution of procedure A
produces B or C or D or E which covers all possible
outcomes.)
Chapter XV
The Language of a Model

As a first approximation we define the language of a Model to be the Model with its non-logic axioms omitted, leaving the formal language of its formal theory and its procedures, experts and community. One then has sentences given meaning by the procedures and theorems deduced from the logic axioms. (We later discuss “meaning” in more detail in chapter XVII.) To develop this idea of a Model’s language we refine our notion of a procedure as follows.

In the above discussion of procedures we probably conveyed the idea that a basic predicate in a formal Model such as the distance between points x and y is z could be a procedure. What is the distance between the two front corners of my desk? Take out a tape measure and measure it. For a Model only dealing with distance measured with a tape measure, this is fine. But suppose the Model also includes statements such as, “The distance between the earth and the moon is 240,000 miles,” that is, distance measured in a very different way. The full force of the theorems of the Model may be required to justify using “distance” in these two ways. To get around this we amend our definition of procedures by requiring them to be specific. Experts agree that when a procedure is executed twice, in each case one does the same thing (where “same” is defined by executing the procedure Executions x and y are the same).

We amend our definition of a language by including lexicographic axioms as part of the logic axioms, for example, the tape measure distance between x and y is z, implies the distance between x and y is z, in which case the distance between x and y is z is a predicate but not be a
procedure. Also include as axioms all lexicographic definitions of the usual sort, defining a word in terms of previously defined words and the logic words.

Constructing a model from a unit of knowledge involves a variety of choices and the unit of knowledge may be much influenced by the perspectives of the unit’s creators and more broadly by its community. As a first step in addressing dependence of knowledge on viewpoint, that is, on relativity of knowledge, we take up translating the language of one model into another’s language. We define a translation from language $L_1$ to language $L_2$ to be an assignment of each noun of $L_1$ to a noun of $L_2$, preserving type of noun and to each basic predicate of $L_1$ a basic predicate in $L_2$. Recall we specified that all languages have the same logic symbols and logic axioms. Hence since predicates are built out of basic predicates and logic symbols all predicates will translate into predicates in $L_2$. Furthermore a translation is required to take names of procedures into names of procedures and the experts of the two languages agree that the execution outcomes of procedures and their translations agree. Define a translation between two languages as translations in both directions.

We now have a method of comparing models, namely, by translations between their underlying languages obtained by throwing away their non-logic axioms.
Chapter XVI
Relativity of Knowledge

Dependence on point of view is completely pervasive in our interactions with the world. Our interactions always depend on our personal, social and cultural vantage points from which we make sense of and give meaning to these interactions. The question is how, and how successfully, do we correct for this dependence.

Starting with the basics, a simple but characteristic example of the way we deal with correcting for point of view is provided by the way we deal with the weight of objects, “This is a pound and a half steak.” Our everyday dealing with the weight of objects involves a point of view expressed as units of measurement, pounds or grams, for example. We are perfectly used to the idea that a physical object has weight even though we need a unit in order to measure it and say what its weight is. When one says something weighs ten pounds we are announcing a point of view, pounds, and its weight from this point of view. In an isolated culture using pounds, this could go unnoticed. Confronted with another culture using grams, some might declare that weight is subjective nonsense or a cultural artifact. A more subtle approach is to say that the weight of an object is a number and a unit of measurement (Our culture has an institution, the Bureau of Weights and Measures, which had a standard one pound object and sold copies of it.) For this to make sense the following condition must be satisfied: If you tell a gram man this book weighs 10 pounds, he needs to know that a one pound object weighs 453.6 grams, from which he knows that if he weighed the 10 pound book he would get 4536 grams.
In the case of weight, as with most measurements, we have no difficulty in correcting for dependence on viewpoint, represented by the need to choose a unit, and we have no difficulty in thinking of an object as having a weight, independent of a choice of unit. We attempt to formulate this idea more precisely. Think of measuring the weight of an object A as the outcome of a procedure which says, A has weight \((w, U)\), where \(w\) is a number and \(U\) is an object, \(U\) being thought of as specifying the unit of measurement (for pounds, \(U\) would be a one pound object, possibly the one held by the Bureau of Weights and Measures) and \(w\) the number giving its weight in \(U\) units. If \(U\) weighed one pound we could say A weighs \((2.5, U)\) instead of, A weighs 2.5 pounds. In addition we have an idea of two such pairs being equivalent; for example, if \(P\) is a one pound object and \(G\) is a one gram object \((10,P)\) and \((4536,G)\) are equivalent. Someone saying, “this is \((10,P)\) bag of flour” is the same as saying, “this is a \((4536,G)\) bag of flour.”

Fractions provide a somewhat more complicated example of this idea with \(2/3 = 4/6\). Is there a number out there, possibly in Plato’s ideal world with “\(2/3\)” and “\(4/6\)” both names for it? Should we have been writing “\((10,P) = (4536,G)\)” instead of “\((10,P)\) and \((4536,G)\) are equivalent”.

It turns out that in some situations it is most convenient to use the equivalence approach, “\((10,P)\) and \((4536,G)\) are equivalent” and in others, a platonic approach, “\(2/3 = 4/6\)”. We use both, noting that it is easy to switch from one to the other. Using the Platonic approach, for example, replacing “\((10,P)\) and \((4536,G)\) are equivalent” to “\((10,P) = (4536,G)\)” where “\((10,P)\)” and “\((4536,G)\)” are names for the same thing.
In order for the use of “equivalence” to work, as above, it must satisfy the following conditions:

A is equivalent to A.

If A equivalent to B, then B equivalent to A.

If A equivalent to B and B equivalent to C, then A is equivalent to C.

Having their own way of dealing with “being”, mathematicians worked out a precise way of sorting out the above applied to fractions. The fraction a/b is the collection of all pairs of natural numbers c and d such ad = cb. Then 2/3 = 4/6 (Two sets are equal if they have the same elements.)

Different languages also provide examples of dependence on point of view and translations provide the method of correcting for this dependence. We address the question of the relativity of knowledge by using this idea of translating one language into another to compare different units of knowledge. To simplify the discussion we continue to assume all our models contain the same formal rendering of logic.

Define two Models, $M_1$ and $M_2$, to be strictly equal, written $M_1 = M_2$, (or strictly equivalent) if there is a translation between their languages taking axioms into theorems in both directions; this implies theorems go to theorems in both directions. The communities and experts may be different; recall procedures are parts of a language and are preserved under translation. Think of $M_1$ and $M_2$ as different frameworks from which to view an aspect of the world, rather like different units for measuring the same thing, feet and meters for measuring length. Although an actual $M_1$, Maxwell’s electro-magnetism for example, had various equivalent formulations and may be convenient among equivalent frameworks, but epistemologically, it is
an arbitrary choice, just as feet is an arbitrary choice of a unit for measuring length.

A more interesting way of comparing Model’s is empirical equality, which we define as follows. Recall a verified procedure is a procedure which has all its outcomes theorems and all its executions have produced theorems. Define two Models, $M_1$ and $M_2$, to be *empirically equal* if there is a translation between their underlying languages taking verified procedures to verified procedures in both directions. One can then have two Models which are empirically equivalent but may have contradictory axioms, something which happens in practice during the development of a subject or with social knowledge with different communities and ideologies.

One may check that strictly equal and empirically equal are based on equivalence relations. Using empirical or strict Model equality, we can now formulate notions of *viewpoint-free knowledge*, that is, absolute knowledge. Just as $(10, \text{pounds})$ and $(4536, \text{grams})$ can be viewed as the weight of some object, two empirically equal Models can be viewed as absolute empirical knowledge about some aspect of the world. The aspect of the world a Model is applicable to is characterized by the Model’s procedures. Of course the importance attached to a procedure or its Model can vary wildly from culture to culture and over time. Analogous definitions and remarks can be made using strict Model equality. Absolute empirical knowledge is probably the important variety for science and engineering while empirical and strict Model equality is important for social knowledge.

Although utilizing our Model prospective for a thorough examination of Postmodernism’s claims concerning science and the relativity of knowledge is beyond the scope of this
paper, we note that most of these views are based on “continental philosophy” which places thought or consciousness at the heart of epistemology with only remote contact with the rest of the world. From the perspective of our Models, the doctrines of continental philosophy are like mathematics in that they have only the formal language part of knowledge and hence are much closer to mathematics than to experimental science. Lyotard ([4]) succinctly captures these views with his statement: “To the extent that science does not restrict itself to stating useful regularities and seeks the truth, it is obliged to legitimate the rules of its own game.” But discovering and stating those regularities is what science is all about. Those regularities manifest themselves in the outcomes of procedures. Aggressively pursuing those regularities has transformed our world.”
We return to a general discussion of our Model approach setting forth views of “meaning”, “facts” and “laws of nature”. For the unsophisticated, the meaning of "apple" is surely related to the procedure "x is an apple", the meaning of “apple” being illuminated when "x is an apple" is executed, in which case attention is drawn to only one apple. One “gets the idea” of apple from a number of executions of the procedure and from executions of other procedures such as "x is an orange". But giving a precise meaning to “apple” is much like the problem of inductive reasoning giving empirical truth. As a Model evolves, possibly confronting questions such as “Is this an apple?”, new procedures are added which refine the meaning of its words, for example in response to genetic manipulations. From the perspective of Models, formalizing "x is an apple", the meaning of “apple” may be taken as the procedure x is an apple, emphasizing that meaning requires context and may be non-linguistic.

Meaning for the sentences and words in the language of a Model comes from the lexicographic definitions of the language, that is, words defined in terms of other words, and from the procedures imprecisely giving meaning to the undefined words, i.e an execution of x is a straight line for the meaning of straight line. Procedures illuminating lexicographically defined words, by way of examples, are also vital. But complete precision of meaning is lacking and is probably undesirable as being too constraining.

Probably an important part of the meaning of sentences is a relation between sentences which we name “refinement” and define by p’ is a refinement p if p’ is a plausible answer
to the question, “What did you mean by saying p?” Usually there are several possible answers having different meanings from one another. A precise meaning may involve an infinite regression as is possible in Socratic dialog, going from signifier to signifier without ever reaching the signified. In practice, of course, the process usual stops after a few steps when the questioner is satisfied that he has enough for action.

Another meaning of “meaning” is an answer to a question asking the meaning of an event. A Model for which the execution of some of its procedures provide a description of the event could be taken as it’s meaning in as much as it provides a context for viewing the event.

Largely due to the revolution in logic initiated by Gottlob Frege, analytic philosophy has been preoccupied with questions about linguistic meaning, the way language is related to the world. In contrast with finding meaning in peoples minds, a common approach to meaning, Frege ([11]) associates to each part of a sentence - noun, predicate, etc. - its “reference”, intuitively what it stands for. For example, the reference of “George Washington” is a certain man. The references of the parts of a sentence then combine, in a systematic way, to determine the truth or falsehood of the sentence. It turned out that he needed to refine “reference” by adding to a part of a sentence, a “sense”, roughly the way a reference is referred. These admittedly obscure notions can be rendered in an obscure form in our Model setting.

Our Model of a unit of knowledge can be viewed as including an approach to “meaning” similar to Frege’s. In the Frege approach to the sentence “This tangerine weighs ¼ pound” can be divided into two nouns, “this tangerine” and “¼”, and one predicate “x weighs y pounds”. A Model
for this could have nouns, **this tangerine** and ¼ and a procedure **x weighs y pounds**. When the procedure is executed it produces **this tangerine weighs ¼ pound or not this tangerine weighs ¼ pound**. The meaning of the sentences’ parts, “this tangerine”, “¼” and “x weighs y pounds” could be the procedures **is the tangerine, is ¼ pound, and x weighs y pounds**, that is, ways of interacting with the world. In general we can take “meaning” to be procedures in Models. If the weight Model above is to include weighing my automobile, **x weighs y pounds** may or not may not be a procedure. Weight might be divided into, kitchen scale weight, and, 500 pounds to a 10,000 pounds weight scale, each of which could give rise to procedures or or further refinement and possibly identifiable as Frege’s “senses”.

Philosophers sometimes say a proposition about the world is true if it corresponds, or is verified, by the facts! But what is a fact? We define a fact using an equivalence relation the way we did with fractions. We define a fact to be a pair (f,M), where M is a Model and f is a sentence in the language of M; a point of view M, and an assertion f, from that point of view. Furthermore, (f,M) = (f',M'), with respect to the following equivalence relation: Pairs (f,M) and (f',M') are equivalent (may be called equal) if and only if, M and M' are empirically equivalent by a translation, translating f into f'. Furthermore, we require that there is an (s,M) in which f was produced by executing a procedure of M.

By the same maneuver one may define a law of nature” as we did with pairs using strict equality and requiring f to be a theorem instead of the outcome of a procedure. Although in science, dealing with a variety of viewpoints is usually avoided by making an historically convenient
choice, the existence of different viewpoints is not entirely philosophical. In theoretical physics, showing that two points of view are equivalent is sometimes a major accomplishment. With the advent of a new idea, within months, a large number of papers will come out setting forth various ways of viewing it.
Chapter XVIII
Kuhn and Postmodernism

In Thomas Kuhn’s *The Structure of Scientific Revolutions in Science* ([3]), scientific research that extends the applicability of a generally accepted and perfected theory to previously unexplored situations is called “normal science”, “puzzle solving”. Replacing a previously accepted theory by a significantly different theory is a “revolution”– Ptolemy to Copernicus, Newton+Maxwell to Einstein, Einstein to Einstein+quantum mechanics. Although he offers various factors that might influence choosing one theory over another, he seems to say that in revolutions, the choice of which theory is accepted as valid from competing theories is not decided by experimental evidence, a claim much utilized by postmodernist to justify the notion that each body of knowledge has its own notion of truth applicable only to itself and that notions such as predictive power is simply not applicable outside of science. What Kuhn actually says is more complicated than this but not easily sorted out. In his 1969 “clarifications”, he writes,

There is no natural algorithm for theory-choice, no systematic decision procedure which, properly applied, must lead each individual in the group to the same decision. …What one must understand, however, is the manner in which a particular set of shared values interact with the particular experiences shared by a community of specialists to ensure that most members of the group will ultimately find one set of arguments rather than another, decisive.

This is a sociology and/or history of science claim that I feel no need to address in as much as there is a
A straightforward historically accurate algorithm for replacing theories all scientists agree on, which we next describe.

A prediction, in what was judged to be a well understood situation, turns out to be wrong in some circumstances. Or more commonly, efforts to broaden a Model’s applicability may require revisions. Someone falsifies our boiling water model by discovering that on a mountain top water boils at a temperature well below 212°F. The model is then altered to accommodate this discovery as well as extending the procedures to take altitude into account: on a mountaintop water my boil at 160°F.

The researcher in pursuing normal science tests Boiling-Water under new circumstances. Should Boiling-Water still work, Kuhn would call this normal science. Should the experimenter discover that on the top of a mountain the temperature of the boiling water was 160°F, he knows that Boiling-Water has a problem. He would not be so foolish as to stop using Boiling-Water in situations were it had previously worked. Nevertheless revolution is now in the air with Nobel Prize potential. One scientist comes forward with a theory that thermometers read low at high altitudes, which is dismissed by pressing an ice cube to a thermometer while carrying it up the mountain. A flurry of experiments of various sorts are executed and various theories are put forward. Finally a new theory is accepted. Boiling-Water is renamed Boiling-Water I and modified to produce Boiling-Water II by adding the postulate in

$$\forall x \ (x \text{ is water}) \implies ((x \text{ has temperature } 212^\circ \text{F}) \text{ if and only if } (x \text{ is boiling}))$$

to
∀(x) (x is water) implies(∀(y) ((altitude is y) implies ((x is boiling) if and only if (temperature x = T(y))))

and T(y) is a function specified in the model. Boiling-Water III is Water I with is boiling replaced by is boiling at sea level. From I to II is a Kuhn revolution; I and II logically contradict one another and their conceptual frameworks diverge, boiling temperature has changed from a constant associated with a substance to a dynamic variable. In Kuhn’s terminology I and II are incommensurable, the models contradict one another while I and III are equivalent, which Kuhn denies on the grounds that is boiling in I and is boiling at sea level in III are semantically different.

A more complicated example is Kuhn’s assertion that the “truth” of Newtonian Mechanics and Einstein’s Relativity theory cannot be compared, there is only truth from the standpoint of each; relativity is not an improved version of Newtonian Mechanics, it is an incomparably new way of looking at the world. The transformation from Newtonian Mechanics to Einstein’s Relativity theory, although much more complex, is of the same sort as the above boiling water example. Although popular accounts of Relativity emphasize astronomy where, until very recently it had no effect, it reconciled Newtonian Mechanics and Maxwell’s equations governing electricity and magnetism, solving a major problem of the day. It turns out that Newtonian Mechanics’ predictions of the kinetic energy of fast moving particles are way off. It’s like water boiling well below 212ºF on mountaintops.

Of course finding that water boiled at 160º on a mountaintop might be taken calmly. Discovering that Newtonian Mechanics was seriously wrong was much more
wrenching. People learn new things by grafting them onto their existing knowledge. Grafting the speed of light is independent of the observer, onto Newtonian Mechanics, is traumatic, and certainly deserving of Kuhn’s description of it as being revolutionary. (As shocking as having a boy on a tricycle pedals past you at six miles an hour. You run after him but however fast you run, he still moves away from you at six miles an hour.)

Within our Model setting we offer a counter argument to Kuhn’s claims with respect to the incomparability of Newtonian Mechanics and Relativity. Define a Model $U$ to be an empirical extension of a Model $U'$, if $U'$ is empirically equivalent to a Model contained in $U$, and define strict extension in the same way. We claim that Relativity theory is an empirical extension of Newtonian Mechanics and can with a little tinkering be made a strict extension as follows: Suppose $N$ and $E$ are the models for Newtonian Mechanics and Relativity theory, $T$ is the procedures associated to $N$, say in 1890, and $R$ is the procedures for $E$ in 1940. Then the combination, $(E, T)$ is a sub-Model of $(E, R)$ and $(E, T)$ is empirically equivalent to $(N, T)$. That is, the verified, and the verifiable predictions of $(N, T)$ and $(E, T)$ are the same.

We offer a simple general criteria for comparing Models: Model $A$ is better than Model $B$ if $A$’s predictive procedures include all of those of $B$’s and $A$ has some not shared by $B$. During the making of a choice between $A$ and $B$ a certain amount of turmoil goes on, but in the end the better of the two is chosen. This is certainly the case in the revolutions sighted by Kuhn.

If $A$ and $B$ share predictive procedures and each has some not shared, keep them both. We do not recklessly through away power, and in the fullness of time $A$ and $B$ may be subsumed under $C$. Although all of the elements of
our Model formulation are contained in Kuhn’s work, our main criticism of Kuhn is he is analyzing science from the vantage point of the humanities culture; he implicitly focuses on the “truth” of theories.
Chapter XIX
Platonic and Thales Forms of Knowledge

The Thales and Platonic Forms of knowledge, as described in Chapter I, are knowledge of, respectively, the physical world – animal, vegetable and mineral – covering a vast complex subject about which a small but increasing amount is known by humans, and the spiritual-social part of the world, also vast. In both cases knowledge of these two worlds are used by people to deal with their problems and manage their lives. Until recently, in the West, the Old Testament, the Christian Bible and Aristotle’s works were taken to be pretty much all there was to be known, aside from earthy practical matters, making the Platonic Form of knowledge all that was needed for serious knowledge. The rise of science, since about 1700, accounts for the vast expanse of knowledge in the Thales Form and a substantial portion of the knowledge in the Platonic Form being transformed and moved into the Thales Form, for example, the history of man over the last 60,000 years.

Our Model prospective on units of knowledge provides a way of delineating and comparing Platonic and Thales Forms of knowledge. In our formulation, the Thales Form largely encompasses non-social practical knowledge about the world extending to engineering and the experimental sciences, more precisely specified by our notion of a Model of a unit of knowledge from the physical world (Chapter XI). Although Plato seems to be much taken with geometry, his serious concerns were with social knowledge: morality, social structure, government and the good life. We here take Platonic Form to mean Plato and his followers’ view of knowledge and more generally, much of the present day
views in the humanities and portions of the social sciences, more or less covered in Chapter XI by Models for the units of knowledge from the “social world”.

From our model perspective, the two turn out to be strikingly different from one another. The big difference being that man can and does very drastically modify his spiritual–social world, in contrast to his modest abilities to change the physical world. The difference is huge. For example, the possibility of humans changing the speed of light is absurd. In contrast, the strict social control of when a woman is allowed to become pregnant, strictly enforced from the beginning of the development of agriculture, has in a few years been entirely abandoned by a large fraction of the population of the United States.

Although the differences between the Platonic and Thales Forms are not precise, sharing as they do many features and being matters of degree, the emphases are very different. In a Model for a unit of knowledge in the Thales Form, the theory is rigidly controlled by the world via outcomes of its procedures. Although socially embroidered, most food crops must be planted at certain times, times that humans largely cannot modify. Furthermore, within empirically equivalent renderings, a Model is essentially independent of its community; the relation is the world controls the Model, with the exceptions of the Model changing the world arising from the community's choice of applications of this type of knowledge, for example, the production and use of penicillin, the government regulating and supporting public health, the development of TV. Another feature of knowledge in the Thales Form, what society knows, theories, procedures and predictive power, grows over time. Old predictive power, that is, in familiar circumstances where the outcome of the execution of certain
procedures are predictable, may be exceeded but not contradicted nor lost. Progress, in the sense of more knowledge, aside from social concerns, is unequivocal.

In many ways the Platonic Form of knowledge is the opposite of the Thales Form. The theory largely controls the social world by constraining and directing human behavior. Most notably in modern times, in the Thales Form, changes of theories are expected and welcomed. In the Platonic Form, theory and to some extent procedures, change over time, sometimes very rapidly, but theory and procedures are viewed as largely permanent. Areas of change are very restricted. Claims of progress are often questionable. Often postmodernists put knowledge that is in the Thales form, for example science, into the Platonic form producing a variety of absurd assertions.

In contrast with science, in the Platonic Form, a statement being a theorem (possibly an axiom), that is, “proved” in the formal model, may confer great authority on it. In contrast, in the Thales Form, the power of a theorem is largely contingent on its predictive power. Mathematical models are constructed and from them predictions are deduced. For example, in fluid mechanics, a branch of applied mathematics, theorems yield a design for the cross section of an airplane wing. This design has been used as a starting point in designing actual airplane wings. Such designs are of course tested in wind tunnels and actual airplanes and appropriately modified if needed. The fact that the design is the conclusion of a theorem in fluid mechanics theory only suggests that it is worth looking at. Being a theorem gives a statement very little authority, except in mathematics. In contrast with most ideologies, for example, Free Enterprise doctrines or Marxism, no one passionately insists that airplane wings must use the fluid mechanics
design because it can in some sense be proved. Marx thought in the Platonic Form way, via Hegel, and erroneously equated “scientific” with non-religious accounts of human history.

In mathematics and more generally, in science, a strategy for finding a proof that a statement is a theorem consists of alternatively trying to prove it and to disprove it, logically or empirically, often by trying to find a counter example. In contrast, units of knowledge in the Platonic Form are rarely subject to the critical analysis commonly used in Thales Models. For example, Kant’s a’ priori statement concerning morality is justified on the claim that a’ priori statements exist and, granting that, the one he states, by a simple argument, must be one.

Sorting out the portions of a society's legal system between the Platonic and Thales Forms is complex. At any particular time a portion of law can be largely characterized by a model formulation, theory and practices in executing the laws, placing it in a Thales Form, but nevertheless, very much a social construction that over time may drastically change. For example, the first ten amendments of the Constitution, having been passed, were largely ignored and seriously violated until about 1920, and thereafter, very much utilized. Change in the law, over time, is often in response to changing morality and both morality and the law, over time, are very much in the Platonic Form. There are a variety of theories of law built on setting forth systems of first principles, as in Euclid, by philosophers of law, John Rawls being a recent example. Perhaps, for the law, a hybrid model would be more appropriate, over all a Platonic Form with time dependant Thales sub-models and the overall model containing the mechanisms (higher courts?) for modifying these sub-models.
Major parts of both law and religion are in the Platonic Form, attaching very great importance to proving the truth of some assertions by deducing them, using logic, from a very large text taken to be true. Inasmuch as from a logic point of view, one starts with a vast collection of axioms: sacred religious texts, a body of law, which are unlikely to be consistent, and hence any statement and its negation can each be proved, giving great flexibility to applications of such knowledge. Actually, such large texts are better viewed as part of the world with procedures for accessing and using them. The procedure then is to select as axioms several statements from the large body of axioms and argue from them, with of course two such sub-models possibly contradicting one another. There may be systems of preference to resolve such conflicts, for example national law trumping state law and more often chosen ideological preferences. In addition to logical arguments, extending and modifying the meanings of words and hence modifying the axioms, is of major importance.

The U S Supreme Court provides an interesting example of Platonic arguments. On the occasion of the appointment of a new member, much is made of the notion that there is simply the law, and that the justices should simply apply it to the cases before them and not create new law. The law with respect to a particular case is out there perhaps in the Plato’s ideal world, only to be discovered.

An interesting contrast between Platonic and Thales Forms is provided by history and cosmology (history of the physical universe.) Neither of them can perform experiments on their subject matter. Both history and cosmology can and do make predictions based on theory that can be confirmed by experiments, searching of documents for history and telescope observations for
cosmology. The accounts of the history of the universe are firmly in the Thales Form, constructing an account of the past based on reconciling well established principles of physics and data obtained from telescope measurements. This program assumes the physical principles and the universal constants, for example the speed of light, do not change over time, although there would be no censuring of exploring the possibility that the constants do change over time. It shares with most hard science basic research in having no applications of interest or use in the community at large, while such research of this basic useless type is largely responsible for the massive material transformation of human life over the last few centuries.

Comparing cosmology with history, telescopic data can be replaced by documents and archaeological information, but the physics is replaced by the conceptual framework by which a society views itself, which changes very substantially over time. Our historical models of the past tend to be constructed largely by projecting back in time present social structure similar to early anthropologists who tended to simply fit the primitive societies social structure into a present western framework.

The dichotomy of Platonic and Thales Forms of knowledge is probably relevant in the argument of creationism versus evolution. From the Thales vantage point, the power of the observation that evolution is only a theory, is perhaps diminished by noting that the “sun will rise tomorrow” is also “only” a theory. Creationism is in the Platonic Form while evolution is in the Thales Form and has great predictive power there. The enthusiasm for teaching both in a single high school course would probably be diminished if such a course included, as it should, the importance of experimental evidence, predictive power and
applications of Darwinian evolution, in contrast with the benefits provided by Creationism. For most biologists, whether Darwinian evolution is precisely true in some absolute, timeless sense is not interesting.

We list some features of the Platonic Form, which play an important role in the humanities and religion.

(1) Knowledge consists of true statements about the world, where world is taken in the broadest sense.

(2) Until very recently, that God exists should be included among the potentially true statements and preferably turn out to be provably true.

(3) More recently, various religious-political doctrines, Kantian philosophy, Hegelian dialectical idealism, dialectical materialism, free-enterprise, claim to be true by their adherents, on Platonic Form type grounds, that is, by virtue of deep thinking and logical argument.

(5) In their more pristine form, a doctrine consists of a collection of clearly true statements, the postulates, and logical deductions from them. Great weight is given to propositions that can be proved to be true.

(6) Language is fully used with its concepts about the world, their interrelations and their connections to the material world. “This is an apple.”, coupled with holding up certain objects, plays a role. But the doctrines are not crucially confounded by observations and interaction with the world, save with important documents. Acquaintance with the world and deep thinking provides the basis for truth, which is elaborated through logical arguments.

In the Thales Form, essentially, predictive power replaces the above properties 1 through 6. It is the basis for the human project of trying to control the natural world for human purposes, largely in contrast with understanding and shaping human institutions.
Chapter XX
Concluding Comments

We briefly return to our Historical Fiction of Chapter I extending our time frame to be from the hunter-gatherers to the present day. From the earliest recorded times, human communities developed theories and practices for dealing with their members’ and their communities’ problems and desires. Based on accounts of human life in the distant past and from anthropologists’ studies of primitive societies, a plausible conjecture is that up to very recent times, human societies have theories and practices for dealing with all of life’s problems and desires. No problem or desire, no matter how distant from an efficacious response, fails to have a theory and an associated remedy - doing something is better than doing nothing. From our perspective, this is most notable in present primitive societies, see for example Malinowski’s “Magic Science and Religion” based on his studies early in the 20th century of Trobriand Islanders. Perhaps this human proclivity of addressing problems with theories and practices is part of our Darwinian evolution.

Our combining of “serious knowledge”, largely social, and “practical arts” into one category, called knowledge, may date from the Renaissance. The combining of science and engineering mostly occurred in the 18th century. For example, in the development of the steam engine, over a period of a hundred years, commencing in 1704, the technology of steelmaking and the accurate crafting of steel, combined with the invention and engineering development of the steam engine, a development made possible by the joining of engineering and the science of heat (thermodynamics), produced the Watt steam engine and the
development of a steam engine powerful enough and light enough, in weight, to pull a railroad train.

The splitting off of religion as a separate subcategory of knowledge probably dates from the 19th century and remains a category difficult to define. Although features of one religion, say Christianity, can be found in other cultures, defining a concept of “a religion” has largely eluded social scientists.

Probably early on, and certainly by the time of the Greeks, few things passing for knowledge, fail to be expressible in a language, fail to use the words “and”, “or”, “not”, “implies”, “for all” and “for some”, satisfying the usual axioms, fail to have sentences designated as true, at least in some formal sense, and perhaps recognize that contradictions among the true sentences is to be avoided. Contact with the world, the material, natural, spiritual and human world, is at least realized through some system of communal naming of things in some moderately consist way; all agree, “This is an apple”, “This painting is in the Rembrandt style”, possibly mediated through some notion of “concepts”. That is, most bits of knowledge have informally a theory in our Model sense and the theory is connected to the world via interactions with the world intertwined with names of things in the world and predicates about aspects of the world. In addition, the designation of being “true” may be constrained by the outcomes of human interactions with the world, what we described as predictive power. A good deal of knowledge offers ways of thinking about aspects of the world and influences human behavior without actually saying anything about the world that can be verified, that is, may lack verifiable procedures and of course “truth” is not relevant for a good deal of what is designated as knowledge. Some verifiable procedures may
make successful predictions with their success depending on all members of the community responding in certain ways. Finally there do seem to be statements about the world that are verifiable, have been repeatedly verified and are regarded as true in all cultures at all times. “Night follows day”, “Water can be made to boil by heating it.” They would seem to be absolute knowledge of the world. Our claim is that most practical and scientific knowledge is absolute in this sense. This is not to say that it is easy to assess whether some part of knowledge “really” makes claims about the world, especially for knowledge directly involving human affairs.

As we defined it, a Model of a unit of knowledge, is a combination of a theory, procedures, experts and community, which can be associated to most units of knowledge in a reasonably unambiguous way capturing a good deal of its defining features and in consequence the theorems of the Model carry over as “truths” of the unit of knowledge.

Returning to the relativity of knowledge from the vantage point of our Models approach, we certainly agree that the formal theory and a Model’s set of procedures, taken separately, are meaningless, that meaning and verifiability of a Model are notions within the framework of a community and that Models are social constructions dependent on particular communities and their cultures. Our claim is that this cultural dependence may be of very little relevance as to whether a particular Model is significantly related to “reality” or the extent to which it has authority for all cultures. The cultural relativity of a bit of knowledge is something that can be assessed - of course by a process admitting of further refinement and lacking certainty. It is useful to imagine two communities with different cultures
coming together. They may share equivalent forms of some Model’s and not others. How would they sort this all out?

We have emphasized two main axes of variation of Models, the extent to which a Model is constrained by its predictive procedures, which varies between having no predictive procedures to being fully constrained by them, that is, being a mature Model. A second axis is the extent to which its predictive power is dependent on the communities culture. Although cultural dependence is easily assessed in many cases but it can be very difficult, for example, in the social sciences.

Speaking generally, most would concede that knowledge very significantly involves language, language plus something else. We have emphasized formal languages because of their crispness and power to set forth reasoning in an unequivocal way free of metaphysical baggage. But there is a saying about the limitations of systems such as formal languages: You must put truth in to get truth out. Historically speaking, the “something else” has been an ideal world of ideal forms, thought, consciousness, sensory data, all possible worlds, human social interaction, and we have emphasized planned, aggressive interaction with the world. Our claim is that formal languages plus interaction with the world give a significant part of a unit of knowledge and, in the case of science, a plausible epistemology most scientists would agree with.

Perhaps a useful category of knowledge generalizing Plato’s vision would be Deductive Sciences, which would include mathematics and parts of the humanities, most notably philosophy and theology. A great deal of discussion would go on in this category involving mixtures of science and the humanities. The work in this category would largely be developing conceptual frameworks.
As to the “truth” of statements about the physical and social worlds, our Model formulation provides formal (logic) truths, the outcomes of procedures of Models and theorems in mature Models (empirical truth). Adding “social truth” would seem to complete the question and cover:

- The sun rises each morning.
- God exists.
- Water can be made to boil by heating it.
- Thou shall not bear false witness.
- \( E = mC^2 \)
Addendum I
Set Theory
(Having labored through logic it would be sinful not to touch on set theory)

The sorting out of logic and the associated notion of a formal language, as presented in Chapters III and IV, was a major foundational achievement of the late 19th and earliest 20th centuries. Although of slight relevance to the main line of our arguments, in this addendum, we give a brief account of Set Theory, which when combined with logic, forms the generally accepted foundation of modern mathematics, Russell and Whitehead’s, Principica Mathematica being a rendering of this combination (See [1] for a present, improved version). According to the Encyclopedic Dictionary of Mathematics, “The notion of set introduced toward the end of the 19th century, has proved to be one of the most fundamental and useful ideas in mathematics.” Curiously, with notable exceptions, mathematicians are the only ones who know about this development.

Around 1880 Georg Canter began the development of set theory, a simple idea that turned out to be a remarkable subject. Some collections of things, such as a set of dishes or a dozen eggs, qualify as being things in their own right; they can be subjects in sentences. The intellectual jump was to make any collection of things an object. Any specified collection of things can be designated as a set and given a name, which grammatically, is a proper noun. The set consisting of Ohio, Plato and today’s weather can be the subject of a sentence; “The set consisting of Ohio, Plato, and today’s weather is happy.” is grammatically ok, as is, “My
mother’s set of good dishes has a gravy boat.” Also, Plato is an element of the set consisting of Ohio, Plato, and today’s weather and yesterday’s weather is not. Woodrow Wilson is an element of the set consisting of all American presidents, which, using the standard symbol “ε” for “is a element of” can be rendered as “Woodrow Wilson ε the set off all American presidents.” The symbol “ε” is read as “is an element of” or “is a member of”. If N denotes the natural numbers (the set of natural numbers), 27 ε N, as is 39,478 ε N. By way of notation, the “set consisting of Ohio, Plato, and today’s weather”, may be replaced by “{Ohio, Plato, today’s weather}” (the symbols { and} say ignore the order of the members of the list) and then Plato ε {Ohio, Plato, today’s weather}

With the aim of reducing the truths of mathematics to logic truths, Russell and Whitehead, in Principica Mathematica, took set theory as part of logic and presented logic as a formal theory, as in Chapter IV. It has only one prepredicate ε and x ε y is read as “x is an element of y”. This formal theory could, in our language, be called “formal set theory”. The axioms for ε are complex ([1]) but roughly consists of:

(1) If A(x) is a predicate, then
∃y(∀x x ε y if and only if A(y)) (There is a set x consisting of all things y such that A(y) is true.)

(2) The axiom of choice, which we describe below.

Suppose you are in a store with a large number of sets of dishes on view and you form a new set by choosing and putting into your shopping basket one piece of china from each set of dishes. The axiom of choice says you can do this. Furthermore, it says that you can do this for any collection of non empty sets, that is, given any collection of non empty sets, you can form a new set by choosing an element from
each of the given sets. It was not known whether this was a theorem or not, somewhat similar to the parallel postulate in geometry. It seemed plausible, but yielded some strange corollaries including a number of very surprising, powerful consequences. For example, like the natural numbers, 1,2,3,…, the real numbers can be put in order so that every real number has a closest success. In the early years of the twentieth century a number of prominent mathematicians refused to use it, but objections soon faded. The situation was similar to the parallel postulate in geometry. Is it a theorem or worse, is its negation a theorem. In 1965 Paul Cohen proved that it was neither, in the standard set theory. Most mathematicians chose to add the axiom of choice to the axioms of set theory, as Euclid chose to include the parallel axiom in geometry.

Another curious result in set theory is that there are an infinite number of distinct infinites. A subset of a set S is a set all of whose elements are elements of S. From a set S one can form a new set, called its power set by taking the new set’s elements, all the subsets of S. A standard way of constructing the real numbers from the natural numbers consists of doing the combination of power set and subset, six times.

Recall, a cardinal number says “how many”, 1 or 2 or 3 or … . A set with only a finite number of elements has a cardinality 1 or 2 or 3 or… . It turns out that a sensible definition of cardinality can be defined for any set, adding infinite cardinals to the usual ones 1, 2, 3, … . Furthermore, the cardinality of the power set of a set is always larger than cardinality of the set. The cardinality of the power set of a finite set with n element is 2^n; the cardinality of the natural numbers is the smallest infinite cardinal; the cardinality of the power set of the natural numbers is equal to the
cardinality of the real numbers, and on it goes to ever increasing infinites. A famous conjecture, called the continuum hypothesis, is that there are no cardinal numbers between the cardinalities of natural numbers and the real numbers. Paul Cohen proved that neither it nor its negation were theorems in the standard formulation of set theory.
Addendum II
Gödel’s Incompleteness Theorem

In contrast with Set Theory, Gödel’s incompleteness results have generated a vast literature in the humanities, discussed not only by philosophers, mathematicians, and logicians, but by theologians, physicists, literary critics, photographers, architects, and also have inspired poetry and music ([2]), especially a curiosity in that the discussions of it in the humanities are largely based on a misunderstanding of the meaning of the incompleteness theorems.

In 1931 Kurt Gödel announced his famous incompleteness theorem: any formal theory (as in chapter IV), which includes a formal rendering of the natural numbers, contains a sentence which is not a theorem nor is its negation a theorem. Shouldn’t any statement about something as simple as the natural numbers be true or false and hence provable or disprovable.

For our discussion, we take as an example formal theory, the formal theory, as in chapter IV, with subject axioms the formalized (bold faced) Peano postulates for the natural numbers. As a possible sentence that it and its negation may not be theorems, we take the Goldbach conjecture, which has not been proved to be true or false; it says that any even natural number greater than 3 is the sum of two prime numbers, 4=2+2, 6=3+3, 8=5+3, … . It certainly has the feel of something that is true or false. For any given number, determining that it is the sum of two primes only requires skill in long division and the conjecture has been verified up to a large number. Of course, the conjecture is that it is true for all natural numbers. In the
fullness of time mathematicians may prove that it is true, that is, present a proof broadly accepted by the mathematics community. This acceptance would, by way of checking its logical correctness, include convincing evidence that the proof could be successfully formalized, that is, it is a theorem in an appropriate formal theory. This is in contrast with giving the conjecture and a random formal theory including the whole numbers. The mechanics of setting up a formal theory influences which of its sentences are undecidable! The setting up of a framework from which to explore a particular mathematical area has always been part of the advance of mathematical knowledge.

Byway of assuaging a possible dimming of the enthusiasm in the humanities over the incompleteness theorem, we note, as mentioned in Addendum I, it has been proved that choice is a choice.
Bibliography
