The Algebra of Dynamic Aggregate Demand and Aggregate Supply

These notes present an algebraic version of the dynamic aggregate demand and aggregate supply model presented in Chapters 21 and 22 of Cecchetti *Money, Banking, and Financial Markets*.

<u>Modern Monetary Policy and the Dynamic Aggregate Demand Curve</u>

We begin with Chapter 21 equation (3)

$$\begin{array}{ccccccccccc} \textit{Aggregate Demand} & = & \textit{Consumption} & + & \textit{Investment} & + & \textit{Government Purchases} & + & \textit{Net Exports} \\ Y^{ad} & = & C & + & I & + & G & + & NX \end{array} \tag{1}$$

Following the standard presentation, we will examine consumption and investment individually. Consumption by households depends on their income after taxes – their disposable income. Assuming that there is some minimum, subsistence level, of consumption, we can write this as

$$C = \bar{C} + a(Y-T), \quad 0<a<1 \tag{2}$$

where $\bar{C}$ is subsistence consumption, Y is income, and T is taxes. The coefficient a is the marginal propensity to consume out of an additional dollar of disposable income, and is less than one. (To the extent that consumption varies with consumer confidence, it will show up has changes in $\bar{C}$.)

To keep things clean and simple, assume that investment is the only interest rate sensitive component of aggregate demand. We know from the discussion of Chapter 21 that we expect investment to rise when the real interest rate falls. Denoting r as the real interest rate, we can write this as

$$I = \bar{I} - br, \quad b>0 \tag{2}$$

where b measures the sensitivity of investment to the real interest rate, and $\bar{I}$ is the portion of investment that depends on other things (like business confidence).

Finally, we assume that government purchases G and net exports NX, as well as taxes T are determined outside of the model, so we write them as

$$G = \bar{G}, \ NX = \overline{NX}, \ \text{and } T = \bar{T} \tag{3}$$

Substituting (3) and (2) into (1), and noting that aggregate demand Y^{ad} equals income Y (the Keynesian cross from principles of economics):

$$Y = \bar{C} + a(Y - \bar{T}) + \bar{I} - br + \bar{G} + \overline{NX} \tag{4}$$

Solving for Y, yields the following expression:

$$Y = \frac{\bar{C} + a\bar{T} + \bar{I} + \bar{G} + \overline{NX}}{1 - a} - \frac{b}{1-a} r \tag{5}$$

This is an IS curve.

To make things a bit easier, we will define the first term in the IS curve (5) to be $\bar{Y}$ and the coefficient on the real interest rate r to be b', so that

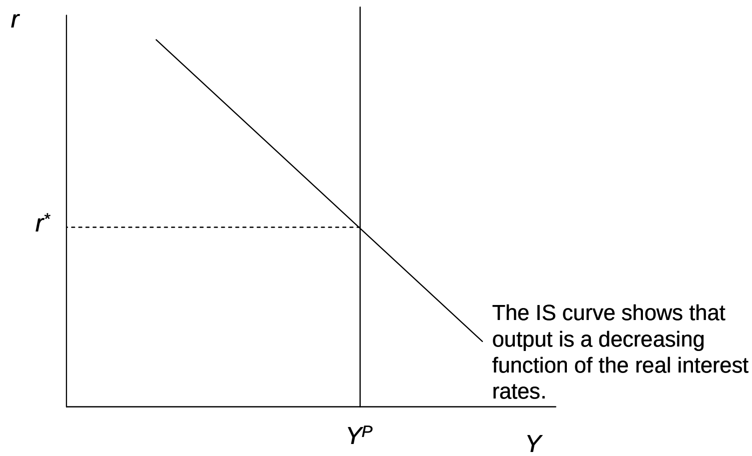
$$Y = \bar{Y} - b'r \tag{5'}$$

Recall from Chapter 21 that the long-run real interest rate r^* is the level that equates aggregate demand with potential output. Writing potential output as Y^P, this means that

$$Y^P = \bar{Y} - b'r^* \tag{6}$$

Or

$$r^* = \frac{\overline{Y} - Y^P}{b'} \qquad (7)$$



Derivation of the long-run real interest rate as the interest rate on the IS curve consistent with potential output.

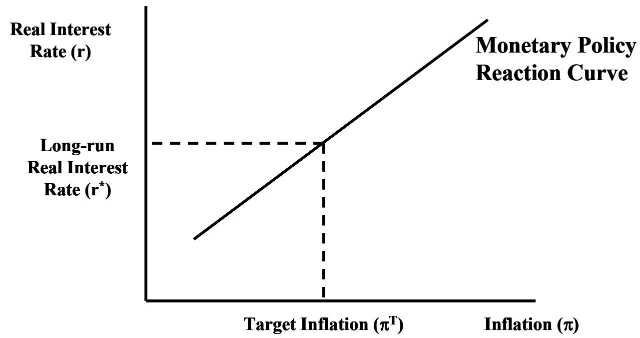
Following the development in Chapter 21, the next step is the introduction of the monetary policy reaction curve – the relationship between the real interest rate set by policymakers and deviations of inflation from the inflation target, π^T. This is

$$r = r^* + \beta(\pi - \pi^T) \qquad (8)$$

where β measures the sensitivity of the slope of the MPR – the sensitivity of deviations of movements in the real interest rate to deviations of inflation from the target.[1] The top half of Figure 21.4 shows the MPR and is reproduced below.

[1] The Taylor rule introduced in Chapter 18 adds a second term related to the output gap, so (8) becomes $r = r^* + \beta(\pi - \pi^T) + \gamma(Y - Y^P)$. This complicates the algebra of the modeling without adding additional insights. The only point to note is that with the addition of this term, the coefficient β no longer needs to be positive for the model to be stable. More on this below.

Monetary Policy Reaction Curve



Monetary policymakers react to changes in current inflation by changing the real interest rate. Increases in current inflation lead them to raise the real interest rate, while decreases lead them to lower it. The monetary policy reaction curve is located so that the central bank's target inflation is consistent with the long-run real interest rate that equates aggregate demand with potential output.

To continue with the derivation of the dynamic aggregate demand curve, substitute (8) into (5') to get

$$\begin{aligned} Y &= \overline{Y} - b' [r^* + \beta(\pi - \pi^T)] \\ &= (\overline{Y} - b' r^*) - b'\beta(\pi - \pi^T) \end{aligned} \tag{9}$$

Notice a few important things about equation (9):

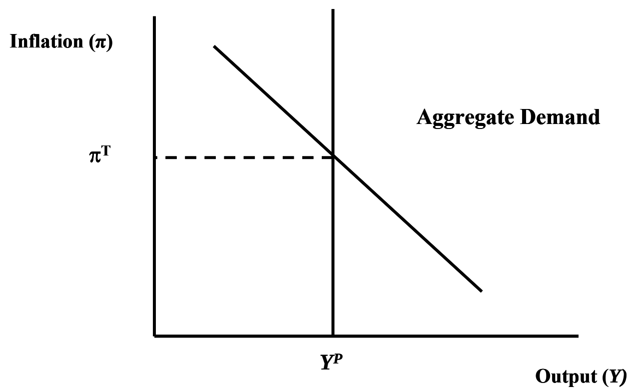
1. It is downward sloping – increases in inflation reduce aggregate demand.
2. Changes in components of aggregate demand that are not sensitive to the interest rate change $\overline{Y}$ and shift the IS curve, and so can have an impact on aggregate demand.
3. Shifts in the monetary policy reaction curve arising from changes in either the long-run real interest rate r^* or the central bank's inflation target π^T, shift the aggregate demand curve as well.
4. The steeper the monetary policy reaction curve, the larger β, the flatter the aggregate demand curve.

The final step in deriving the dynamic aggregate demand curve is to substitute the long-run real interest rate expression (7) into (9). The result is

$$(\pi_t-\pi^T) = -\frac{1}{b'\beta}(Y_t - Y^P) \tag{10}$$

where we have added time subscripts that will be useful later. This equation tells us that the current output gap (Y_t-Y^P) is negatively related to the current deviation of inflation from its target $\beta(\pi_t-\pi^T)$. The result is drawn in the following figure:

The Dynamic Aggregate Demand Curve



At higher levels of inflation, policymakers raise the real interest rate, driving interest sensitive components of aggregate demand down.

An important implication of this is that changes in $\bar{Y}$ do not have any impact on the location of the dynamic aggregate demand curve. The reason is that changes in $\bar{Y}$ compensate for changes in the long-run real interest rate. That is when $\bar{Y}$, r^* changes, shifting the monetary policy reaction curve in a way that leaves the dynamic aggregate demand curve unaffected. Importantly, this is true of any aggregate demand shift (or "shock").

As described in the text, the algebra makes clear that the dynamic aggregate demand curve shifts have no impact on the equilibrium. The reason is that policymakers are assumed to realize immediately what has happened, and compensate for it by shifting their monetary policy reaction curve to reflect the new long-run real interest rate. The only things that shift the dynamic aggregate demand curve are permanent changes in the inflation target π^T and shifts in potential output, Y^P.

Aggregate Supply

Turing to aggregate supply, in Chapter 22 we noted that the short-run aggregate supply curve (SRAS) is flat at the current level of inflation, but that deviations of current output from potential output shift the SRAS up and down. In addition, we noted the possibility of inflation shocks that push inflation up or down temporarily. Writing this all algebraically:

$$\pi_t = \pi_{t-1} + \alpha(Y_{t-1}-Y^P) + e_t\ ,\ \ \alpha>0 \tag{11}$$

where e_t is the inflation shock, which is usually zero, and the coefficient α determines the extent to which inflation responds to the output gap.

Equation (11) tells us that inflation today depends on inflation yesterday and the output gap yesterday. A positive out gap drives up inflation and the SRAS tomorrow; while a negative output gap forces inflation and the SRAS down tomorrow.[2]

Equilibrium

Setting dynamic aggregate demand equal to aggregate supply means solving (10) and (11) for the output gap and inflation. For inflation, the result is

$$(\pi_t-\pi^T) = (1-\alpha\ \beta b')(\pi_{t-1}-\pi^T)+ e_t\ . \tag{12}$$

Because the output gap is proportional to the deviations of current inflation from its target (see equation 10), this means that the expression for the output gap has exactly the same form as (12). We can write it as

$$(Y_t-Y^P) = (1-\alpha\ \beta b')\ (Y_{t-1}-Y^P) - \beta b'\ e_t\ . \tag{13}$$

There are a few things to notice about equations (12) and (13). First, there is the fact that inflation shocks move output and inflation in opposite directions (so long as the dynamic

[2] We could make the model more complicated by adding a term in the current output gap. This would give the SRAS some slope. While this is somewhat more realistic, it merely complicates the exposition.

aggregate demand curve slopes down, which is the case whenever $b'\beta>0$). If, for example, $e_t=+1$, then $(\pi_t-\pi^T)=+1$ and $(Y_t-Y^P)= -\beta b'$. Notice, however, that the extent to which the inflation shock influences output depends on the slope of the monetary policy reaction curve, β. The steeper the MPR, the bigger β, the flatter the dynamic aggregate demand curve and the bigger the fall in output for a given inflation shock.

Another important thing to note about these expressions is that stability of the economy requires that the coefficient $|1-\alpha \beta b'| <1$. If it isn't, then the economy will not be self-correcting. (Under normal circumstances this expression is both positive and less than one, insuring a nonoscillatory return to the long-run equilibrium.)

Looking at the solution, we see that following a unit inflation shock ($e_t=+1$) output and inflation adjust as follows:

Time Period	Inflation Deviations from Target $(\pi_t-\pi^T)$	Output Gap (Y_t-Y^P)
0	+1	$-\beta b'$
1	$(1-\alpha \beta b')$	$-(1-\alpha \beta b')\beta b'$
2	$(1-\alpha \beta b')^2$	$-(1-\alpha \beta b')^2\beta b$
3	$(1-\alpha \beta b')^3$	$-(1-\alpha \beta b')^3\beta b$
T	$(1-\alpha \beta b')^t$	$-(1-\alpha \beta b')^t\beta b$

As β rises, the MPR gets steeper, and the smaller $(1-\alpha \beta b')$, so the shorter lived the shock will be. The more aggressively the policymakers react to deviations of inflation from the target, the bigger the impact on output and the less persistent the effect of the shock on output and inflation.

Extension I: A More General Monetary Policy Reaction Curve

We can replace the simple equation (8) with the more complex Taylor rule in which the real interest rate depends on both the deviation of inflation from its target and the output gap. That is

$$r = r^* + \beta(\pi-\pi^T) + \gamma(Y-Y^P) \qquad (8')$$

Using this in place of equation (8), the dynamic aggregate demand curve (10) becomes

$$(\pi_t - \pi^T) = -\frac{(1+b'\gamma)}{b'\beta}(Y_t - Y^P) \qquad (10')$$

Using the aggregate supply curve (11), we can solve for the equilibrium expressions:

$$(\pi_t - \pi^T) = \frac{(1+\gamma b' - \alpha\beta b')}{1+\gamma b'}(\pi_{t-1} - \pi^T) + e_t\,. \qquad (12')$$

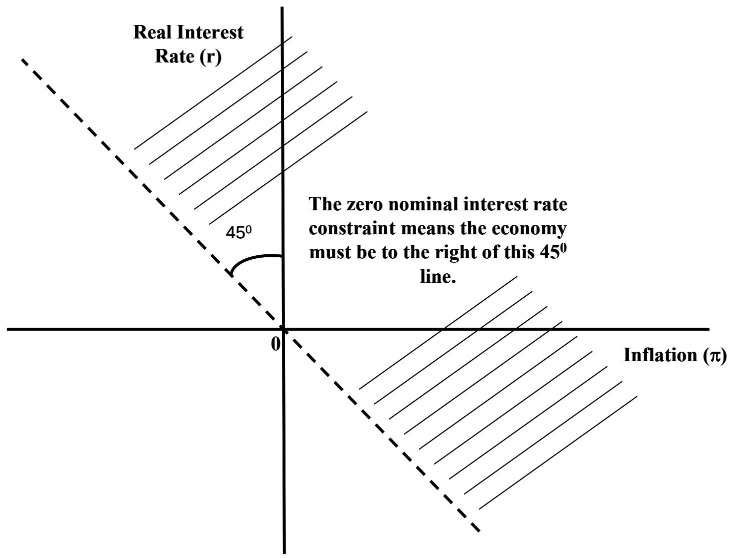
The stability condition requires that the coefficient on lagged inflation be less than one in absolute value – that is the condition for the dynamic aggregate demand curve to slope down and the economy to be self-correcting. Note that this can happen even if $\beta<0$.

Extension II: The Case of Deflation

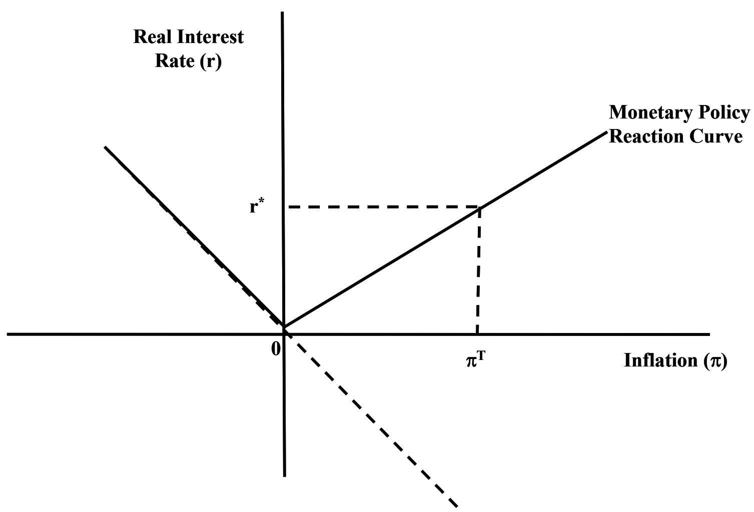
We can use the model to examine the problems caused by deflation that are discussed in Chapter 23. While it is possible to do this algebraically, it is somewhat easier to do it graphically. Before getting to deeply into the discussion, we need to note that deflation only becomes a problem when the economy hits the zero nominal interest rate constraint. As soon as that happens, policymakers can no longer do what they would like.

With output below potential, and inflation below target, policymakers would like to lower the real interest rate by lowering the nominal interest rate. But when the nominal interest rate is at zero, they can’t. And with output below potential, deflation gets worse. This *raises* the real interest rate, further depressing output, driving prices down even more rapidly.

This means that the monetary policy reaction curve must be on the right hand side of the 45^0 line that goes through the origin. This is the region shown in the figure below.



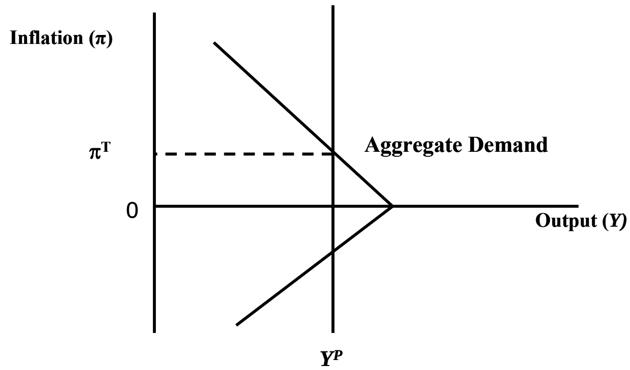
To analyze the impact of deflation the implications of this constraint for the monetary policy reaction curve. Extending the diagram to points where inflation is negative means taking account of this constraint. The result is drawn below. When inflation is near the target, everything is as before. But as inflation falls below zero, the curve starts to slope the wrong way.



As inflation falls, there comes a point where the economy hits the zero nominal interest rate constraint. At this point, the monetary policy reaction curve begins to slope the wrong way as shown by the solid line in the figure

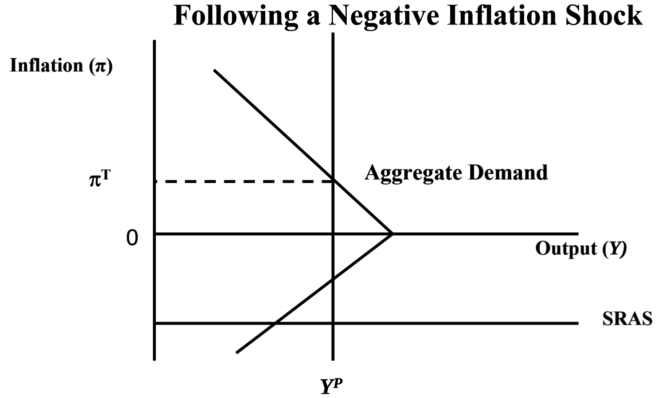
The implications of this for the dynamic aggregate demand curve are profound. When the monetary policy reaction curve slopes down (as it does in the left-hand portion of the above figure) the dynamic aggregate demand curve slopes up. The result is shown below.

The Aggregate Demand Curve with Deflation



Now consider the consequences of a severe negative inflation shock that drives output below Y^P and inflation below zero. The results are shown in the figure below:

The Aggregate Demand Curve with Deflation Following a Negative Inflation Shock



This economy will not move back to Y^P and π^T. Instead, output and inflation will both continue to fall.