

Things to Know for the Physics GRE

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Some ideas borrowed from review sheets by Marty Ligare and Dave Shoepf of Bucknell University, as well as from Wikipedia.

1 Math

1.1 Coordinate systems

Cartesian to Spherical:

$$r = \sqrt{x^2 + y^2 + z^2} \quad \phi = \arctan(y/x) \quad \theta = \arccos(z/\sqrt{x^2 + y^2 + z^2})$$

Spherical to Cartesian:

$$x = r \cos \phi \sin \theta \quad y = r \sin \phi \sin \theta \quad z = r \cos \theta$$

Cylindrical to Cartesian:

$$x = \rho \cos \phi \quad y = \rho \sin \phi \quad z = z$$

Cartesian to Cylindrical:

$$\rho = \sqrt{x^2 + y^2} \quad \phi = \pm \arcsin(y/\rho) \quad z = z$$

Cylindrical to Spherical:

$$r = \sqrt{\rho^2 + z^2} \quad \theta = \arctan(\rho/z) \quad \phi = \phi$$

Spherical to Cylindrical:

$$\rho = r \sin \theta \quad \phi = \phi \quad z = r \cos \theta$$

Spherical:

$$d\vec{l} = dr\hat{r} + r d\theta\hat{\theta} + r \sin \theta d\phi\hat{\phi}$$

$$d\tau = r^2 \sin \theta dr d\theta d\phi$$

Cylindrical:

$$d\vec{l} = d\rho\hat{\rho} + \rho d\phi\hat{\phi} + dz\hat{z}$$

$$d\tau = \rho d\rho d\phi dz$$

1.2 Vector Calculus

Laplacian:

$$\nabla^2 T = \vec{\nabla} \cdot (\vec{\nabla} T) = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}$$

$$\begin{aligned}\nabla^2 \vec{v} &= \nabla^2 v_x \hat{x} + \nabla^2 v_y \hat{y} + \nabla^2 v_z \hat{z} \\ \vec{\nabla} \times (\vec{\nabla} T) &= 0 \quad \vec{\nabla} \cdot (\vec{\nabla} \times \vec{v}) = 0 \\ \vec{\nabla} \times (\vec{\nabla} \times \vec{v}) &= \vec{\nabla} (\vec{\nabla} \cdot \vec{v}) - \nabla^2 \vec{v}\end{aligned}$$

$$\int_a^b (\vec{\nabla} T) \cdot d\vec{l} = T(b) - T(a) \text{ Independent of Path. } \oint (\vec{\nabla} T) \cdot d\vec{l} = 0$$

Stoke's Theorem:

$$\int_{surface} (\vec{\nabla} \times \vec{v}) \cdot d\vec{a} = \oint_{path} \vec{v} \cdot d\vec{l} \text{ (=0 for closed surface)}$$

Green's (Divergence) Theorem:

$$\int_{volume} (\vec{\nabla} \cdot \vec{v}) d\tau = \oint_{surface} \vec{v} \cdot d\vec{a}$$

1.3 Miscellaneous

$$(1+x)^n \approx 1+nx \text{ for } x \ll 1$$

2 Classical mechanics

2.1 Kinematics

$$\text{Force: } \vec{F} = m\vec{a} = \frac{d\vec{p}}{dt}$$

$$\text{Work: } W = \int \vec{F} \cdot d\vec{r}$$

$$\text{Kinetic energy: } T = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

$$\text{Work-Energy Theorem: } W = \Delta T = \frac{1}{2}m(v_2^2 - v_1^2)$$

$$\text{Power: } \frac{dW}{dt} = \vec{F} \cdot \vec{v}$$

Potential Energy:

$$\text{Gravitational: } PE = mgh = -GmM_{\text{Earth}}/r$$

$$\text{Spring: } PE = \frac{1}{2}kx^2$$

Conservative force: $\vec{F} = -\nabla U(\vec{r})$ where $U =$ potential energy

$$\frac{d}{dt}(T + U) = \frac{d}{dt}E = 0$$

$$\text{Impulse: } \vec{I} = \int_{t_1}^{t_2} \vec{F} dt = \Delta \vec{p}$$

2.2 Rotational kinematics

Angular position:

$$\theta = s/r$$

Angular velocity:

$$\omega = \frac{d\theta}{dt} = \frac{v}{r}$$

Angular acceleration:

$$\alpha = \frac{d^2\theta}{dt^2} = \frac{a}{r}$$

Angular momentum: $\vec{L} = \vec{r} \times \vec{p}$ $L_{\text{body}} = I\omega$

Moment of inertia:

$$I = \int r^2 dm$$
$$I = kMR^2$$

where k is given for various objects on the formulas page at the front of the test.

Parallel axis theorem:

$$I_{\text{anywhere}} = I_{\text{center}} + MR^2$$

at a point a distance R away from the center.

Radius of gyration (about an axis):

$$R_g = \sqrt{I/M}$$

Torque:

$$\vec{\tau} = \frac{d\vec{L}}{dt} = \vec{r} \times \vec{F} = I\alpha$$

Kinetic energy:

$$K = \frac{1}{2}I\omega^2$$

Centripetal acceleration:

$$|\vec{a}_C| = v^2/r$$

Tangential acceleration:

$$|\vec{a}_T| = \frac{d|\vec{v}|}{dt}$$

Circular motion:

Period $T = 2\pi r/v$

Centripetal acceleration:

$$|\vec{a}| = \left(\frac{2\pi r}{T}\right)^2 / r = 4\pi^2 r/T^2$$

2.3 Waves and Oscillations

$$f(x, t) = A \sin(kx \pm \omega t)$$

$$k = 2\pi/\lambda \quad \omega = 2\pi/T = 2\pi f$$

Phase velocity: $\omega_{ph} = \omega/k$

Group velocity: $\omega_g = \frac{\partial\omega}{\partial k}$

$$\omega = \sqrt{k/m} \longleftrightarrow \sqrt{\frac{k}{I}}$$

2.4 Simple Harmonic Oscillation

If

$$m \frac{d^2x}{dt^2} = F = -kx$$

then

$$x(t) = A \cos(\omega t + \phi)$$

or alternately

$$x(t) = c_1 \cos(\omega t) + c_2 \sin(\omega t)$$

where

$$\omega = \sqrt{\frac{k}{m}}$$

or, in other words,

$$T = 2\pi\sqrt{\frac{m}{k}}.$$

Consequently,

$$a(x) = -\omega^2 x.$$

Kinetic Energy:

$$T = \frac{1}{2}m \left(\frac{dx}{dt}\right)^2 = \frac{1}{2}kA^2 \sin^2(\omega t + \phi)$$

Potential Energy:

$$U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2 \cos^2(\omega t + \phi)$$

so the total energy is constant:

$$E = \frac{1}{2}kA^2$$

Pendulum of length l (with gravitational acceleration g) for small angle oscillations:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

2.5 Damped Oscillator

For viscous damping $F = -c\frac{dx}{dt}$, equation of motion is

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = 0$$

where $\omega_0 = \sqrt{\frac{k}{m}}$ is the angular frequency of the undamped oscillator.

Damping ratio $\zeta = c/2m\omega_0$ determines behavior:

- Overdamped ($\zeta > 1$): Exponential decay (without oscillation) to equilibrium, slower for higher ζ .

- Critically damped ($\zeta = 1$): Fastest possible return to equilibrium; no oscillation. $C_{critical} = 2m\sqrt{k/m}$
- Underdamped ($\zeta < 1$): System oscillates with amplitude slowly decreasing to zero, with frequency $\omega_1 = \omega_0\sqrt{1 - \zeta^2}$.

2.6 Miscellaneous

Doppler (sound):

$$f' = f \left(\frac{v \pm v_{observer}}{v \mp v_{source}} \right)$$

3 Electricity and Magnetism

3.1 The Basics

Maxwell's Equations:

$$\vec{\nabla} \cdot \vec{E} = \rho/\epsilon_0 \text{ (Gauss)}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \text{ (Faraday)}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \epsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t} \text{ (Ampere)}$$

Coulomb's Law:

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_{volume} \frac{1}{r^2} \rho(\vec{r}') d\tau' \hat{z}$$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_{surface} \frac{1}{r^2} \sigma(\vec{r}') da' \hat{z}$$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_{line} \frac{1}{r^2} \lambda(\vec{r}') dl' \hat{z}$$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{1}{r} \rho(\vec{r}') d\tau'$$

Forces:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

Gauss's Law:

$$\text{Flux } \Phi \equiv \oint \vec{E} \cdot d\vec{a} = Q_{enc}/\epsilon_0 \Leftrightarrow \vec{\nabla} \cdot \vec{E} = \rho(\vec{r})/\epsilon_0$$

Electric field inside a sphere of radius R with uniformly distributed charge Q :

$$E_{inside} = \frac{Q}{4\pi\epsilon_0} \frac{r}{R^3}$$

$$E_{outside} = \frac{Q}{4\pi\epsilon_0} \frac{1}{r^2}$$

$$\vec{\nabla} \times \vec{E} = \vec{0} \text{ always.}$$

Electrical Potential:

$$V(\vec{a}) = - \int_{\circ}^{\vec{a}} \vec{E}(\vec{r}) \cdot d\vec{l}$$

$$V(a) - V(b) = \int_a^b \vec{E}(\vec{r}) \cdot d\vec{l}$$

$$\vec{E} = -\vec{\nabla}V$$

Poisson's Equation:

$$\nabla^2 V = -\rho/\epsilon_0$$

$$\Rightarrow \text{Laplace's Equation: If } \rho = 0, \nabla^2 V = 0$$

Energy of a point charge:

$$W = QV$$

Energy of a collection of n point charges:

$$W_{tot} = \frac{1}{8\pi\epsilon_0} \sum_{j=0}^n \sum_{i=0, i \neq j}^n \frac{q_i q_j}{r_{ij}}$$

Energy of a continuous distribution of charge:

$$W = \frac{1}{2} \int_V \rho V d\tau = \frac{\epsilon_0}{2} \int_{allspace} E^2 d\tau$$

Conductor:
 $\vec{E}_{inside} = \vec{0}$ $\rho_{inside} = 0$ V constant $\vec{E}_{surface} \perp$ surface

3.2 Dielectric Media

Dipole moment:

$$\vec{p} = \alpha \vec{E}$$

where α =atomic polarizability.

Polarization:

$$\vec{P} = \vec{p}/\text{unit volume}$$

Bound charge:

$$\begin{aligned} \sigma_b &= \vec{P} \cdot \hat{n} \\ \rho_b &= -\vec{\nabla} \cdot \vec{P} \end{aligned}$$

Electric displacement field:

$$\begin{aligned} \vec{D} &\equiv \epsilon_0 \vec{E} + \vec{P} \\ \vec{\nabla} \cdot \vec{D} &= \rho_{free} \\ \oint \vec{D} \cdot d\vec{a} &= Q_{free\,enclosed} \end{aligned}$$

Linear Dielectrics:

$$\vec{P} = \epsilon_0 \chi_e \vec{E}$$

where $\chi_e \equiv$ electric susceptibility.

$$\vec{D} = \epsilon_0(1 + \chi_e)\vec{E} = \epsilon \vec{E}$$

where permittivity $\epsilon \equiv \epsilon_0(1 + \chi_e)$

Dielectric constant (relative permittivity):

$$\epsilon_r \equiv 1 + \chi_e \equiv \epsilon/\epsilon_0$$

If linear dielectric fills all relevant space,

$$\vec{E} = \frac{1}{\epsilon_r} \vec{E}_{vac}$$

i.e. replace ϵ_0 with ϵ .

Boundary conditions:

$$\begin{aligned}\epsilon_a E_a^\perp - \epsilon_b E_b^\perp &= \sigma_f \\ V(a) &= V(b)\end{aligned}$$

3.3 Magnetostatics

$$\vec{F}_{mag} = I\vec{l} \times \vec{B} \text{ or } Q(\vec{v} \times \vec{B})$$

Circular motion of a charge in a \vec{B} field:

$$p = mv = qBR$$

Currents:

$$\vec{I} = \lambda\vec{v} \quad \vec{K} = \sigma\vec{v} \quad \vec{J} = \rho\vec{v}$$

Biot-Savart Law:

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{\vec{I} \times \hat{z}}{r^2} dl' = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l}' \times \hat{z}}{r^2}$$

Ampere's Law:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

Solenoid with n turns and current I :

$$\vec{B} = \mu_0 n I \hat{z}$$

Magnetic vector potential:

$$\begin{aligned}\vec{B} &= \vec{\nabla} \times \vec{A} \\ \vec{A} &\parallel \vec{J} \\ \vec{A}(\vec{r}) &= \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}')}{r} d\tau' \\ \oint \vec{A} \cdot d\vec{l} &= \Phi_B\end{aligned}$$

Magnetic dipole moment:

$$\vec{m} = I\vec{a}$$

where \vec{a} is area normal vector.

Magnetization:

$$\vec{M} \equiv \vec{m}/\text{unit volume}$$

H field:

$$\vec{H} \equiv \frac{1}{\mu_0} \vec{B} - \vec{M}$$

$$\vec{\nabla} \times \vec{H} = \vec{J}_f$$

$$\vec{\nabla} \cdot \vec{H} = -\vec{\nabla} \cdot \vec{M}$$

Generally, $\vec{H} \parallel \vec{B} \parallel \vec{M}$.

3.4 Linear magnetic materials

$$\vec{M} = \chi_m \vec{H}$$

where χ_m = magnetic susceptibility.

$$\vec{B} = \mu_0(1 + \chi_m)\vec{H} = \mu\vec{H}$$

where $\mu \equiv \mu_0(1 + \chi_m)$ = magnetic permeability.

$$B_{material} = \frac{\mu}{\mu_0} B_{vacuum} = \mu_r B_{vacuum}$$

where $\mu_r \equiv \mu/\mu_0$ = relative permeability.

3.5 Index of Refraction

$$n \equiv \frac{c}{v_{medium}} = \sqrt{\epsilon_r \mu_r}$$

Speed of light in medium is $v = c/n$.

3.6 Electrodynamics

Electromotive force \mathcal{E} =work done per unit charge

$$\mathcal{E} = \oint \vec{f}_{mag} \cdot d\vec{l} = vBl$$

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = \oint \vec{E}_{induced} \cdot d\vec{l}$$

$$\vec{\nabla} \times \vec{E}_{induced} = -\frac{\partial \vec{B}}{\partial t}$$

$$\oint \vec{E}_{induced} \cdot d\vec{l} = -\frac{\partial \Phi_B}{\partial t}$$

$$\vec{\nabla} \cdot \vec{E}_{induced} = 0.$$

$$\vec{J} = \sigma \vec{E} = \sigma f$$

where σ =conductivity.

Mutual inductance:

$$M = \Phi_{21}/I_1 = \Phi_{12}/I_2$$

Self-inductance:

$$L = \Phi/I$$

$$\mathcal{E} = -\frac{d\Phi}{dt} = -L\frac{dI}{dt}$$

For solenoid with n turns, radius R , height l :

$$L = \mu_0 n^2 \pi R^2 l$$

3.7 Miscellaneous

Doppler (light):

$$f = \left(1 - \frac{v}{c}\right) f_0$$

4 Electronics

Voltage: $V = \int \vec{E} \cdot d\vec{r}$

Power = $Vi = Ri^2 = V^2/R$

Resistors: $V_{across} = iR = R \frac{dQ}{dt}$ (Ohm's Law)

$R_{series} = R_1 + R_2 + \dots$

$R_{parallel} = (R_1^{-1} + R_2^{-2} + \dots)^{-1} = \frac{R_1 R_2}{R_1 + R_2}$ (for 2 resistors)

Capacitors: $V_{across} = Q/C$

$C_{series} = (C_1^{-1} + C_2^{-2} + \dots)^{-1} = \frac{C_1 C_2}{C_1 + C_2}$ (for 2 capacitors)

$C_{parallel} = C_1 + C_2 + \dots$

$E_{cap} = \frac{1}{2} CV^2 = \frac{1}{2} QV = \frac{1}{2} Q^2 / C$

Discharging capacitor: $V_C = V_0 e^{-t/RC}$ $Q = Q_0 e^{-t/RC}$

Charging capacitor: $V_C = V_0 (1 - e^{-t/RC})$

Inside capacitor: $E = \sigma_{free} / \epsilon_0$

Inductor: $V_{across} = L \frac{dI}{dt} = L \frac{d^2 Q}{dt^2}$

To measure R_x , need a voltmeter in parallel with resistance $R_v \gg R_x$ so $R_{eq} \approx R_x$.

Superposition Theorem: The voltage across any two points is (what you get if you open all current sources) + (what you get if you instead short all voltage sources).

Voltage Divider: $V_{AB} = V_{in} \left(\frac{R_2}{R_1 + R_2} \right)$

Current Divider: $i_2 = I \left(\frac{R_1}{R_1 + R_2} \right)$

Kirchoff's Laws: $\sum_{\text{Loop}} V = 0$ $\sum_{\text{Node}} i = 0$

Thevenin's Theorem: The voltage across any two points in a circuit is the same as that across some resistor R_{TH} in series with some voltage source V_{TH} . To find R_{TH} , short V_{in} , find R_{eq} for V_{AB} . Then $V_{TH} = V_{AB}$ and

$$R_{TH} = V_{TH}/I$$

Load: Use a large resistor R_L across A and B; then V_{AB} is as through the resistor isn't there.

Max power dissipated when $R_L = R_{TH}$. Circuit unchanged for $R_L \ll R_{TH}$

Impedance: $Z = v(t)/i(t)$

$$Z_R = R$$

$Z_L = j\omega L$ Large impedance at high freq (coil opposes change in B field). Acts like short at low freq (acts like a wire).

$Z_C = \frac{1}{j\omega C}$ Large impedance at low freq (charge just builds on plates). Acts like short at high freq.

$$\text{Reactance: } X = \text{Re}[Z] \quad X_L = \omega L \quad X_C = \frac{1}{\omega C}$$

Transfer function $H = V_{out}/V_{in}$

Phase $\phi = \arctan(\text{Im}[H]/\text{Re}[H])$

$\Rightarrow V_{out}$ lags by ϕ behind V_{in} .

$$\text{Voltage divider: } V_{out} = \frac{Z_2}{Z_1 + Z_2} V_{in}$$

RC circuit: time constant $\tau = RC$

RL circuit: time constant $\tau = L/R$

LC Circuit (L and C in parallel): $L \frac{d^2 Q}{dt^2} + \frac{Q}{C} = 0 \Rightarrow \omega = \frac{1}{\sqrt{LC}}$

RLC Circuit (Series resonant circuit):

$$|H| = R / \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2} = 1 / \sqrt{1 + Q^2 \left(1 - \frac{\omega_{res}^2}{\omega^2}\right)^2}$$

$$|H|_{max} = 1 \text{ for } \omega_{res} = \frac{1}{\sqrt{LC}}$$

Quality factor $Q \equiv (\text{Power in L}) / (\text{Power in R}) = \omega L / R$

This is a band pass filter: lets through ω_{res} . High $Q \rightarrow$ narrow peak.

Half power points: $H = \frac{1}{\sqrt{2}} |H|_{max}$ at $\omega = \omega_1, \omega_2$. $Q = \frac{\omega_{res}}{\omega_2 - \omega_1}$

Low pass filter: $H = \frac{1}{j\omega RC + 1}$. $|H|_{\omega \rightarrow 0} \rightarrow 1$, $|H|_{\omega \rightarrow \infty} \rightarrow \frac{1}{\omega RC}$
 Corner frequency $\omega_C = \frac{1}{RC}$, $|H| = \frac{1}{\sqrt{2}}$

High pass filter: $H = \frac{j\omega RC}{1 + j\omega RC}$. $|H|_{\omega \rightarrow 0} \rightarrow \omega RC$, $|H|_{\omega \rightarrow \infty} \rightarrow 1$

Tank Circuit: $Z_{eq} = \frac{Z_C Z_L}{Z_C + Z_L} = \frac{j\omega L}{1 - \omega^2 LC}$
 used in band pass filter: $|H| = 1$ for $\omega = \omega_R = \frac{1}{\sqrt{LC}}$

used in band stop filter:

Diode: For forward current, voltage across diode drops by about 0.7 V with 1Ω resistance.

If you are faced with two wires of non-negligible resistance, the thing that's the same about them is $\rho = R * a/l$ where R is the resistance, a the area, and l the length.

5 Thermal Physics

1st Law of Thermodynamics:

$$dE = \delta Q - \delta W$$

that is, the change in internal energy is the heat energy absorbed by the system minus the work done BY the system.

2nd Law of Thermodynamics:

$$dS = \frac{\delta Q}{T}$$

that is, the change in entropy is the heat absorbed by the system divided by the temperature.

Interpretations:

- The entropy of an isolated system that is not in equilibrium increases over time, obtaining its maximum at equilibrium.
- Heat generally does not flow spontaneously from a material of lower temperature to one of higher temperature.
- A cyclic process cannot convert heat into work.

Also, a reversible process must have zero change in entropy.

Helmholtz free energy:

$$F = E - TS$$

where E is the total energy.

Gibbs Free Energy:

$$G = E - TS + PV$$

Specific heats (the heat energy required for a unit temperature increase):

$$\text{At const pressure: } C_P \equiv \left(\frac{\partial Q}{\partial T} \right)_P$$

$$\text{At const pressure: } C_V \equiv \left(\frac{\partial Q}{\partial T} \right)_V$$

Heat energy:

$$Q = mC\Delta T$$

Work:

$$dW = -PdV$$
$$W = - \int_{V_i}^{V_f} PdV$$

6 Quantum Mechanics

Schrodinger Equation:

$$i\hbar \frac{\partial}{\partial t} \Psi(x, t) = H\Psi(x, t) = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \Psi(x, t) + V(x)\Psi(x, t)$$

Time Independent Schrodinger Equation (Eigenvalue equation for H , with standing waves $\Psi(x, t) = \psi_E(x)e^{-iEt/\hbar}$):

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \psi_E(x)}{\partial x^2} + V(x)\psi_E(x) = H\psi_E = E \cdot \psi_E$$

Norm of wavefunction:

$$\langle \psi | \psi \rangle = \int_{\text{allspace}} \psi^* \psi dx$$

Normalized:

$$\langle \psi | \psi \rangle = 1$$

Energy eigenfunctions corresponding to different energy eigenvalues are orthogonal:

$$\int u_E^*(x) u_{E'}(x) dx = 0$$

Dimensions of \hbar : Energy \cdot time. Momentum operator:

$$p\psi = -i\hbar \frac{\partial}{\partial x} \psi$$

Energy of photon:

$$E = h\nu = \hbar\omega = pc$$

where the momentum of a photon is

$$p = h/\lambda = \hbar k$$

Compton Scattering: When a photon of wavelength λ is scattered by a particle of mass m through an angle θ , the change in wavelength is

$$\lambda' - \lambda = \frac{h}{mc} (1 - \cos \theta)$$

Uncertainty Principle:

$$\Delta x \Delta p = \frac{\hbar}{2}$$

Expectation Value:

$$\langle F \rangle = \int_{\text{allspace}} \psi^* F \psi dx$$

6.1 Classic Early Quantum Experiments

Photoelectric effect: • Demonstrated that light is quantized, with quanta of energy $E = hf$

• When electromagnetic radiation above a certain threshold frequency, the radiation is absorbed and electrons are emitted with kinetic energy

$$\frac{1}{2}mv_{max}^2 = E_{kmax} = hf - \phi$$

where h is Planck's constant, f is the light's frequency, and ϕ (or sometimes W) is the Work Function, the electron's binding energy that the photon must possess just to free it from the atom.

6.2 Miscellaneous

Hydrogen atom: Energy levels: $E_n = -13.6 \text{ eV} / n^2$

7 Nuclear Decays

Notation:



A =mass number (number of protons + number of neutrons)

Z =atomic number (number of protons)

Alpha decay: Emission of a $\alpha = {}^4_2\text{He}^{2+}$, so $A \rightarrow A - 4$, $Z \rightarrow Z - 2$.

Beta decay: Emission of β^+ =electron or β^- =positron, along with a neutrino or antineutrino. $A \rightarrow A$, $Z \rightarrow Z \pm 1$

(Think of it as a neutron becoming a proton + electron, or a proton becoming a neutron + positron.)

Gamma decay: Emission of γ =photon. A and Z stay the same.

8 Special Relativity

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Lorentz Transformations for motion along x direction:

$$t' = \gamma (t - vx/c^2)$$

$$x' = \gamma (x - vt)$$

$$y' = y$$

$$z' = z$$

Time dilation: for clock at rest in unprimed system,

$$\Delta t' = \gamma \Delta t$$

Length contraction (length along x direction):

$$\Delta x' = \frac{\Delta x}{\gamma}$$

Energy:

$$E = \gamma mc^2$$

Spacetime interval (Lorentz invariant)

$$\Delta s^2 = c^2 \Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$$

and all observers agree on this value!

Velocity:

$$u'_x = \frac{u_x - v}{1 - u_x v / c^2}$$

9 Laboratory formulas

- If $y = y(x_1, x_2, \dots)$ and each x_i is measured with uncertainty σ_i then the uncertainty in y is

$$\sigma_y = \sqrt{\left(\frac{\partial y}{\partial x_1} \sigma_1\right)^2 + \left(\frac{\partial y}{\partial x_2} \sigma_2\right)^2 + \dots}$$

- When n events (uncorrelated, random) are measured to give a mean with standard deviation σ , the standard error of the mean is σ/\sqrt{n} .

- Reading graphs:

If a log-log graph is linear with slope m , then $y = ax^m$.

If a log-lin graph is linear with slope m , then $y = ae^{mx}$.