Web Appendix for
Interest rate risk and other determinants of
post WWII U.S. government debt/GDP dynamics

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A Reconciling Our Debt Series With Those Reported
by the Bureau of the Public Debt

Much of the analysis in this paper focuses on computing the returns to the marketable debt held by the public. But this is only a portion of the total debt outstanding. In this web appendix, we reconcile our measure of the marketable debt with estimates that include the nonmarketable debt and the debt held within the government.

In table I, we decompose the total Federal debt into its various components as reported by the Bureau of the Public Debt in the Monthly Statement of the Public Debt. The total outstanding debt is partitioned two different ways: 1) marketable and non-marketable; and 2) held by the public and held within the government. At the end of 2009 the total par value of the outstanding Federal debt stood at $12.3 trillion, of which, $7.3 trillion was issued in marketable securities and $5 trillion in nonmarketable securities; $7.8 trillion was held by the public, and $4.5 trillion was held within the government.

After being issued by the Treasury, ownership of marketable securities can be transferred through purchases and sales in the secondary market. Marketable securities are largely made up of Treasury bills, notes, bonds, and Treasury Inflation-Protected Securities (TIPS). In contrast, non-marketable securities are registered to the owner and cannot be sold in

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<table>
<thead>
<tr>
<th>Title</th>
<th>Debt Held By the Public</th>
<th>Government Holdings</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>** Marketable:**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bills</td>
<td>1,787,913</td>
<td>5,567</td>
<td>1,793,480</td>
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<tr>
<td>Notes</td>
<td>4,179,412</td>
<td>1,696</td>
<td>4,181,108</td>
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<td>Bonds</td>
<td>714,672</td>
<td>3,259</td>
<td>717,931</td>
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<td>TIPS</td>
<td>567,851</td>
<td>205</td>
<td>568,055</td>
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<tr>
<td>Federal Financing Bank</td>
<td>0</td>
<td>11,921</td>
<td>11,921</td>
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<tr>
<td><strong>Total Marketable</strong></td>
<td>7,249,848</td>
<td>22,648</td>
<td>7,272,496</td>
</tr>
<tr>
<td>** Nonmarketable:**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Domestic Series</td>
<td>29,995</td>
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<td>29,995</td>
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<tr>
<td>Foreign Series</td>
<td>4,386</td>
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<td>4,386</td>
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<tr>
<td>R.E.A. Series</td>
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<td>0</td>
<td>1</td>
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<tr>
<td>State/Local Govern Series</td>
<td>214,138</td>
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<td>214,138</td>
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<tr>
<td>U.S. Savings Securities</td>
<td>191,298</td>
<td>0</td>
<td>191,298</td>
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<td>Government Account Series</td>
<td>119,932</td>
<td>4,477,200</td>
<td>4,597,132</td>
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<tr>
<td>Hope Bonds</td>
<td>0</td>
<td>492</td>
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<tr>
<td>Other</td>
<td>1,411</td>
<td>0</td>
<td>1,411</td>
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<tr>
<td><strong>Total Nonmarketable</strong></td>
<td>561,161</td>
<td>4,477,693</td>
<td>5,038,853</td>
</tr>
<tr>
<td><strong>Total Public Debt Outstanding</strong></td>
<td>7,811,009</td>
<td>4,500,341</td>
<td>12,311,350</td>
</tr>
</tbody>
</table>

Table I: Summary of Treasury Securities Outstanding, December 31, 2009

Millions of dollars

the secondary market, though typically they can be redeemed on demand. Yields on nonmarketable securities are set administratively, usually by a formula based on the returns for marketable debt. The nonmarketable debt held by the public is largely held by small investors in the form of savings bonds (U.S. Savings Securities) or by state and local governments who by law must hold the proceeds from their own debt issues in Treasury debt until they use the funds. Of the $5 trillion in nonmarketable debt, $4.5 trillion is held in the ‘Government Account Series’, a collection of bonds mostly held by the Social Security trust fund.

While marketable securities today represent the lion’s share of the debt held by the public ($7.2 out of $7.8 trillion, or a little less than 93%), this has not always been the case. In figure I, we plot the debt-GDP ratio for three different measures of the debt: 1) the marketable debt held by the public; 2) the sum of the marketable and the nonmarketable debt held by the public; and 3) the total outstanding debt. Over the entire period, marketable debt has averaged about 80 percent of the total debt held by the public (i.e. the ratio of the solid-blue line to the dashed-red line). Early in the sample, this ratio was about two-thirds, and it has steadily increased over time. Nonmarketable savings bonds and Victory loans played a much larger role in Treasury borrowing during World War II and the Korean War than they do today.

Our focus on the government budget constraint impels us to concentrate on the debt held by the public. In the *Monthly Statement of the Public Debt*, holdings by the Federal Reserve are included under the “held by the public” column. We treat the Federal Reserve holdings as intra-government. In December 2009, the Federal Reserve held $930 billion in Treasury securities or about 7.5% of the total debt outstanding. Currently, excluding the Federal Reserve, the public holds about $6,884 billion (about 56%) of the total debt. In figure I we can see that this share (the ratio of the dashed-red line to dot-dashed black line) has varied from over 80% during World War II to under 50% during the 2000s.

Treasury debt is typically reported at its par value – the face value or value of the principal outstanding. In our analysis we compute the market value – the value of the existing stock of Treasury securities at market prices. In figure II we report the par value and market value of the marketable debt held by the public as percents of GDP. These two series track each other quite closely. However, the ratio of the two series, reported in figure III, shows that prior to 1950 and after 1980 the market value generally exceeded the par value. During the intervening period Treasury securities typically traded on the market for less than their par value.
Figure I: Marketable Debt held by the Public, Total Debt held by the Public, and Total Debt Outstanding as Percentages of GDP

The solid blue line is the ratio of the par value of marketable Treasury securities held by the public to GDP. The dashed red line is ratio of the par value of all Treasury securities (marketable and nonmarketable) held by the public to GDP. The dot-dashed black line is ratio of the par value of total outstanding debt to GDP.
Figure II: Par Value and Market Value of Marketable Debt Held by the Public as a Percent of GDP

The solid blue line is the ratio of the par value of marketable Treasury securities held by the public to GDP. The dashed red line is ratio of the market value of marketable Treasury securities held by the public to GDP.
Figure III: Ratio of the Market Value of Marketable Debt Held by the Public to its Par Value
B Decomposition Plots

In this appendix, we display the contributions to $\frac{\dot{B}_t + \bar{B}_t}{Y_t} - \frac{\dot{B}_{t-1} + \bar{B}_{t-1}}{Y_{t-1}}$ depicted on the right side of equation (4). In figure IV, the top left panel shows $100 \times \sum_{j=1}^{n} \tilde{r}^j_{t-1,t} \frac{\dot{B}_t - 1}{Y_t}$; the top right panel shows $-100 \times \pi_{t-1,t} \frac{\dot{B}_t - 1}{Y_t}$; the bottom left panel shows $-100 \times g_{t-1,t} \frac{\dot{B}_t - 1}{Y_t}$; and the bottom right panel shows $100 \times \frac{\text{def}_t}{Y_t}$. The nominal returns series plotted in the top left panel is the sum of the three series plotted in figure 4 in the paper.
Figure IV: Contributions to Changes in the Ratio of Marketable Debt to GDP

In this figure, the top left panel shows $100 \times \sum_{j=1}^{n} r_j \frac{B_j}{Y_{t-1}}$, the top right panel shows $-100 \times \pi_{t-1,t} \frac{\hat{B}_{t-1}}{Y_{t-1}}$, the bottom left panel shows $-100 \times g_{t-1,t} \frac{\hat{B}_{t-1}}{Y_{t-1}}$, and the bottom right panel shows $100 \times \frac{\text{def}_{t}}{Y_{t}}$. 

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C  Holding Period Returns for Zero-Coupon Bonds

The large swings in the holding period returns reported in figures 3, 8, and 9 in the text can be puzzling initially. In this appendix, we report simple calculations to provide some intuition behind the large capital gains and losses on pure discount bonds. In figure V below, we plot the zero-coupon yield curves from Gurkaynak, Sack, and Wright (2007) for the last business day of the year for 2007 and 2008.

![Zero-Coupon Yield Curves for 2007 and 2008](image)

Figure V: Zero-Coupon Yield Curves for 2007 and 2008

As is well known, yields fell dramatically in 2008. The average maturity of the debt during this period was about 3 years. On December 31, 2007, the yield-to-maturity for a three-year zero coupon bond was 3.0733 so its price was

\[
\frac{100}{(1 + 0.030733)^3} = $89.588
\]

On December 31, 2008 the yield-to-maturity of this same bond was 0.5713 so its price was

\[
\frac{100}{(1 + 0.005713)^2} = $98.867
\]

So the holding period return on this bond for 2008 was

\[
100 \times \frac{98.867 - 89.588}{89.588} = 10.3\%
\]