Gravity, entropy, & entanglement

Brandeis Physics Colloquium

Matthew Headrick

Duality: Two theories that describe the same physics

Hamiltonian mechanics $\longleftrightarrow$ Lagrangian mechanics

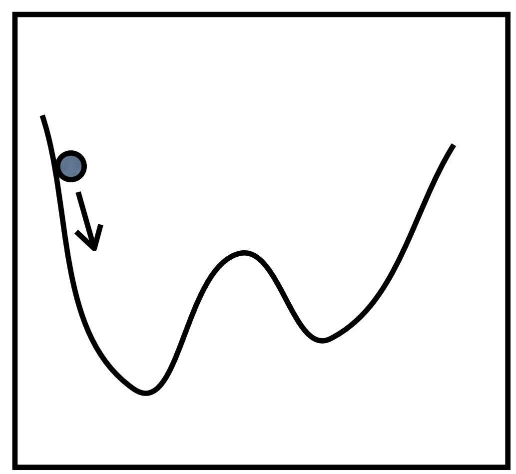
Heisenberg picture $\longleftrightarrow$ Schrödinger picture

canonical quantization $\longleftrightarrow$ Feynman path integral

statistical mechanics $\longleftrightarrow$ thermodynamics

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = 0$$

description A



physical system

$$\dot{q} = \frac{\partial H}{\partial p}$$
$$\dot{p} = -\frac{\partial H}{\partial q}$$

description B

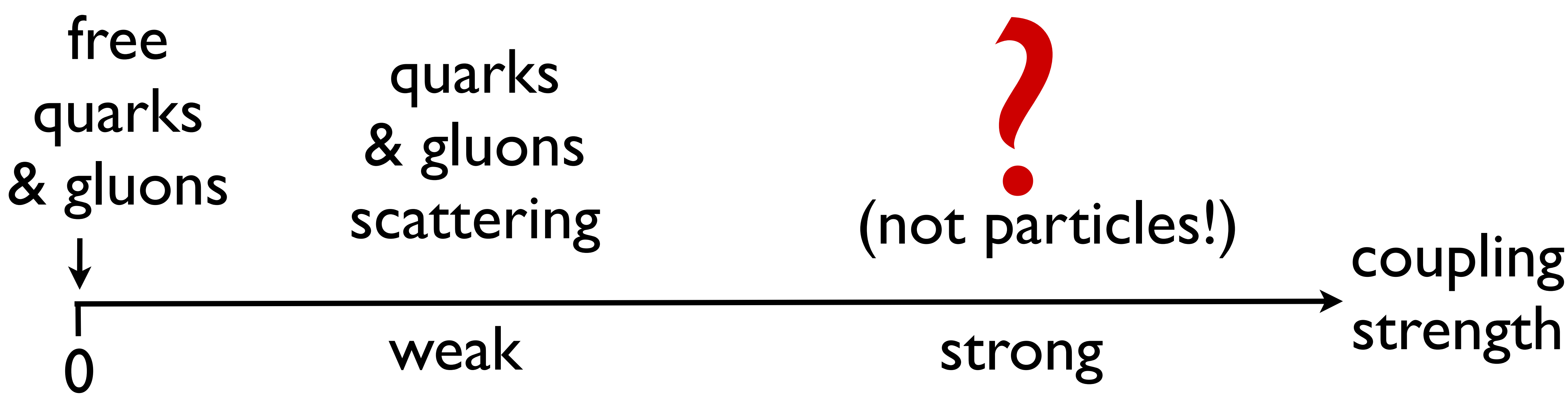
Today: Holographic dualities

One side of the duality: Yang-Mills theory

Example: quantum chromodynamics (theory of strong nuclear interaction):

- quarks: red, green, blue
- gluons: red-antigreen, etc. (8 types total)

These are *fields*: like 8 copies of electromagnetism, with interactions among gluon fields.



‘t Hooft (1970’s): Yang-Mills theories *simplify* when the number of colors N_c is large
(but still no explicit description at strong coupling)

Systems with large number of degrees of freedom often show relatively simple *collective* behavior:

- gas or liquid → fluid dynamics
- crystal lattice → elasticity

Collective description of strongly-coupled YM?

Maldacena (1997): **YES!**

Collective description of strongly-coupled Yang-Mills:

General relativity . . .

. . . with some matter fields . . .

. . . negative cosmological constant . . .

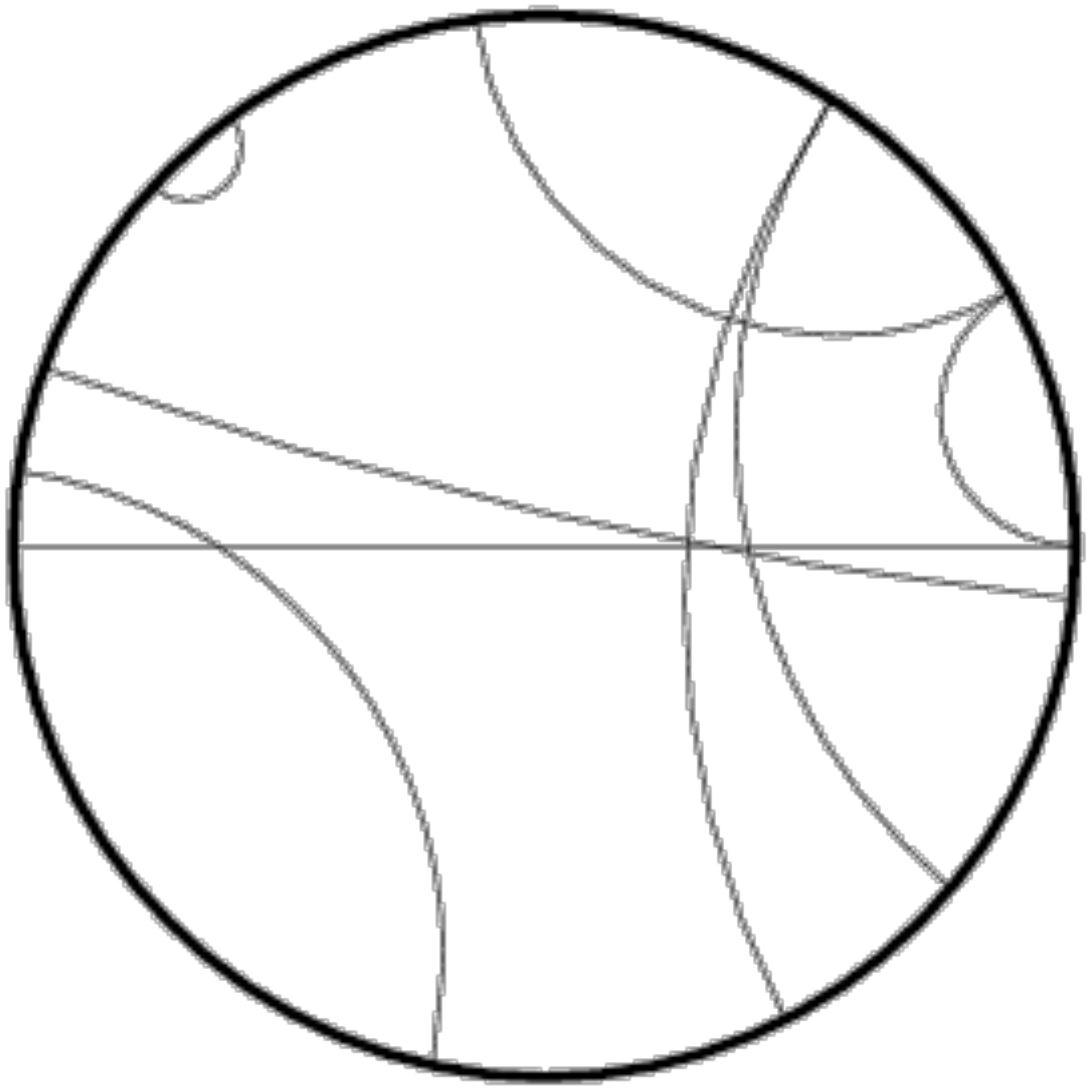
and

. . . an extra spatial dimension

(wait for it)

Huh?

Ground state of GR with $\Lambda < 0$:
anti-de Sitter space (AdS)

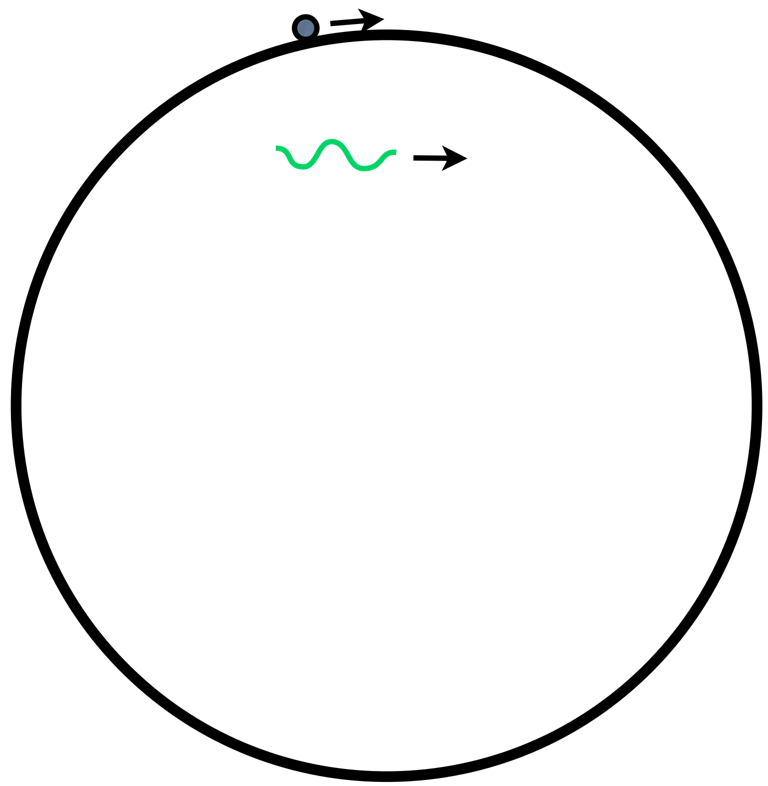


(Escher 1960)

- Walls are infinitely far away
- Gravitational potential attracts matter to center

AdS = vacuum of YM on 3-sphere

Matter or gravitational wave in AdS represents excitation in YM



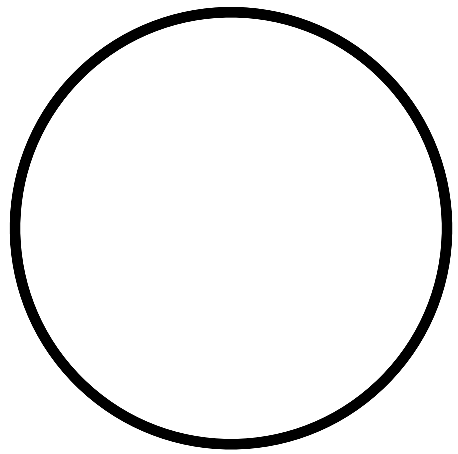
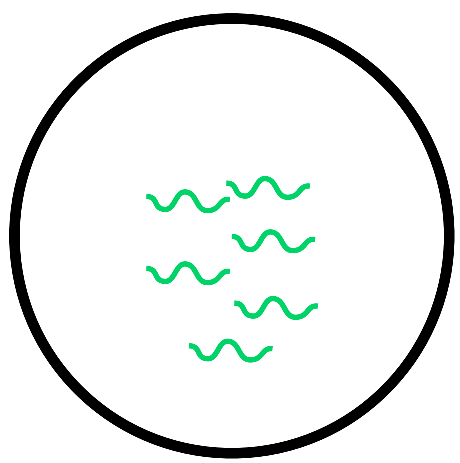
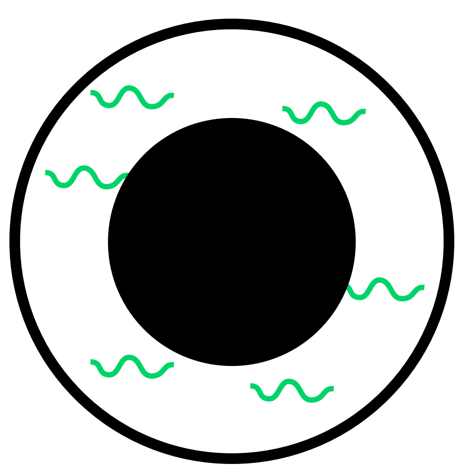
"holography"

$$\frac{R_{\rm AdS}^3}{G_{\rm N}\hbar} = N_c^2 \quad \Rightarrow$$

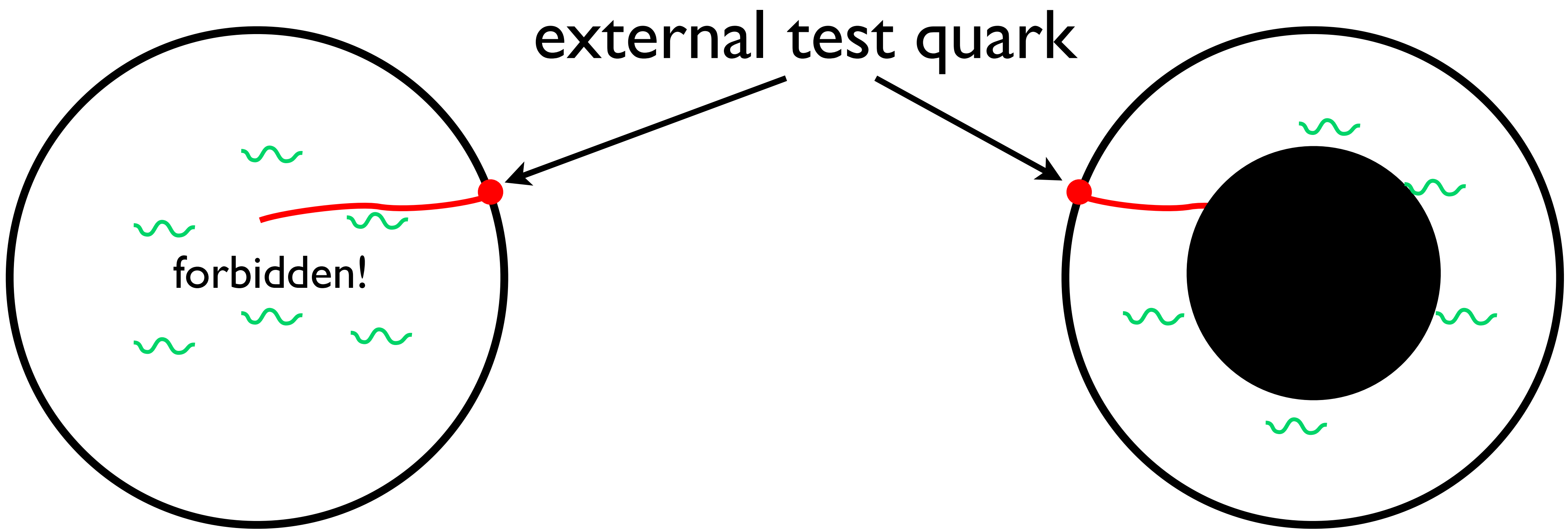
thermodynamic limit $N_c \to \infty$ of YM = classical limit $\hbar \to 0$ of GR

statistical fluctuations in YM = quantum fluctuations in GR

Thermal physics (Witten 1998)

temp	GR	entropy	YM
$T = 0$	 empty AdS	0	vacuum
$T < T_{\mathrm{HP}}$	 gas of gravitons etc.	$\mathcal{O}(1)$	color-confined phase (color singlets excited)
$T > T_{\mathrm{HP}}$	 black hole w/ graviton atmosphere	$\frac{a_{\mathrm{horizon}}}{4G_{\mathrm{N}}\hbar} = \mathcal{O}(N_c^2)$	color plasma (gluons excited)

Confinement vs. screening of color charge



$T < T_{\mathrm{HP}}$ $T > T_{\mathrm{HP}}$

statistical fluctuations in YM

= quantum fluctuations in GR

Example:

Brownian motion of test quark from Hawking radiation

(de Boer, Hubeny, Rangamani, Shigemori; Son, Teaney 2008)

Holographic duality gives us a fundamentally new perspective on gravity:

- tells us that classical gravity is intrinsically a coarse-grained theory
- *defines* quantum gravity
- answers fundamental questions, e.g. Do black holes destroy information?

Also allows us to *compute* in strongly coupled YM, for theory & experiment:

- thermalization & transport (for comparison to quark-gluon plasma of real QCD seen at RHIC)
- quantum-information properties
- hundreds more examples . . .

Thermalization of plasma = black-hole formation, can be studied by classical GR
(only known way to study thermalization in strongly coupled YM)

Find that thermalization can occur faster than expected from naive dimensional analysis
(seen at RHIC) (Chesler & Yaffe; Bhattacharya & Minwalla 2008)

Statistical fluctuations? Out of equilibrium, so fluctuation-dissipation theorem does not hold

We studied onset of Hawking radiation in collapsing spacetime, found fluctuations can also thermalize "instantly" (Ebrahim & MH 2010)

Quantum entanglement: a brief digression

Two systems A, B

Separable state: $|\psi\rangle = |v_1\rangle_A \, |w_1\rangle_B$

Entangled state: $$|\psi'\rangle = \frac{1}{2^{1/2}} \left(|v_1\rangle_A \, |w_1\rangle_B + |v_2\rangle_A \, |w_2\rangle_B \right)$$

To detect entanglement, compute entanglement entropy

Density matrix: $\rho = |\psi\rangle\langle\psi| \,, \qquad \rho' = |\psi'\rangle\langle\psi'|$

Reduced density matrix:

$$\rho_A = \mathrm{Tr}_B \rho = |v_1\rangle\langle v_1|$$

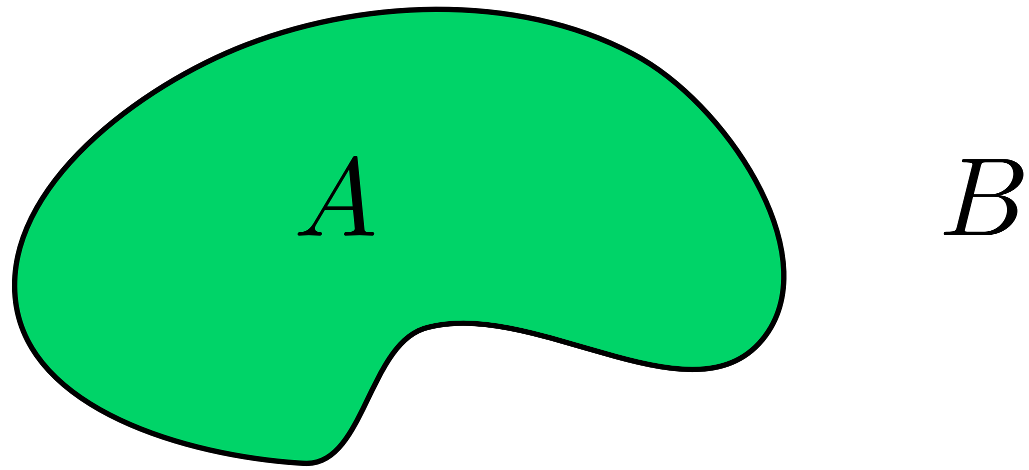
$$\rho'_A = \mathrm{Tr}_B \rho' = \frac{1}{2} \left(|v_1\rangle\langle v_1| + |v_2\rangle\langle v_2| \right)$$

Entanglement entropy:

$$S_A = -\mathrm{Tr}_A \rho_A \ln \rho_A = 0$$

$$S'_A = -\mathrm{Tr}_A \rho'_A \ln \rho'_A = \ln 2$$

In a field theory, A could be spatial region & B the complement

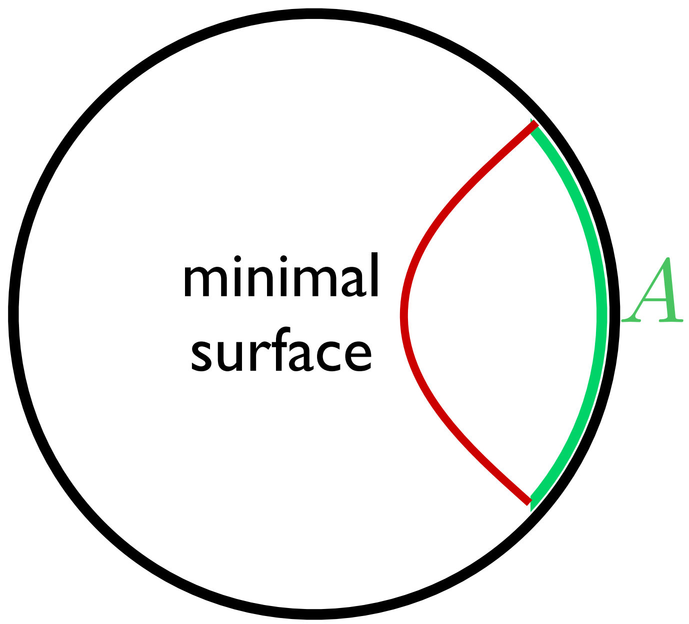


Ground state is generally highly entangled.

How to calculate entanglement entropies in field theories is an important problem

Usually impossible in practice even in *free field theories*

However, in field theories with holographic duals, Ryu & Takayanagi (2006) conjectured a simple (& deep) formula:



$$S_A = \frac{a_{\rm min}}{4G_{\rm N}\hbar}$$

Remains unproven, but passes many tests:

- agreement w/ certain first-principles calculations (MH 2010 & others)
- satisfies strong subadditivity (MH & Takayanagi 2007)

$$S_{A\cup C} + S_{A\cap C} \leq S_A + S_C$$

In progress (Callan & MH): Understanding entanglement entropy in out-of-equilibrium systems.