Statement of Recent Work

One of the most important developments in theoretical physics in this century has been the growing understanding of the role played by quantum entanglement in the physics of extended systems, ranging from condensed matter to quantum gravity. For example, in quantum field theories, entanglement entropies (EEs) and related quantities provide insights that are complementary to traditional quantities such as correlation functions and scattering amplitudes. And in quantum gravity, there is evidence that entanglement plays a central role in the emergence of classical spacetime from underlying quantum degrees of freedom. My work has contributed significantly to the progress made in these areas over the last few years, and in some cases it has created entirely new lines of inquiry. Rather than focusing on particular systems, my investigations have mostly been concerned with trying to understand the general structure, properties, and implications of entanglement in large classes of theories, and developing general calculational methods.

Much of my work has studied the Ryu-Takayanagi formula [1] and its generalizations. This beautiful formula, which represents a vast generalization of the famous Bekenstein-Hawking black-hole entropy formula, relates the EE of a spatial region in a holographic field theory (i.e. a QFT with a classical gravity dual) to the area of a certain minimal surface in the dual bulk geometry. As it was originally the result of inspired guesswork, a focus of much early work was on finding ways to check the formula or to derive it. One of the earliest and most stringent tests was a proof by Takayanagi and myself that the formula obeys the strong subadditivity (SSA) inequality, a fundamental general property of EE [2]. This proof not only provided evidence that the formula is correct, but also suggested that general relativity has some sophisticated quantum information theory built into it. The method of proof, an inclusion-exclusion argument, was novel in this context, and has since found several other applications. For example, Hayden, Maloney, and I showed that the RT formula also obeys another inequality, called “monogamy of mutual information” (MMI) [3]. Unlike SSA, MMI is not a general property of all quantum systems, and therefore reveals that holographic systems have a very special entanglement structure. MMI does, however, imply an infinite set of constrained inequalities that are general properties of EE [4]; in fact, this work established that, remarkably, RT obeys all applicable known general properties of EE. In the paper [5], I collected the known general implications of the formula, proved a few new ones, and discussed their implications for the entanglement structure of holographic theories. More recently, Bao et al. generalized MMI into a new infinite family of inequalities obeyed by RT [6].

A different sort of test for RT is to compare its predictions quantitatively to first-principles calculations of EEs. Such calculations typically involve the so-called replica trick, which requires calculating Rényi entropies (certain generalizations of the von Neumann entropy) as an intermediate step. Early work by Fursaev claimed to derive the RT formula in this way [7]. I showed, however, that Fursaev’s method for calculating Rényis was incorrect, and explained how to do it right [8]; such computations have since become a staple of the field. I also observed that the RT formula predicts a novel type of phase transition in the EE of separated regions as a function of the separation, with the mutual information vanishing beyond the phase transition point. Results of Rényi entropy calculations, which required developing new techniques both from the gravitational and CFT sides, gave evidence in favor of these predictions, and therefore of the RT formula. This work was subsequently pushed forward by Faulkner [9], whose work in turn set the stage for Lewkowycz and Maldacena’s general derivation of RT [10]. In the paper [8], I also initiated the study of entanglement (Rényi) entropies in general (not necessarily holographic) conformal field theories with large central charge; on the basis of these calculations, I conjectured that these entropies are actually the same for all such theories. As a result of this work, large-c CFTs and their EEs have become a major area of inquiry, with notable work by Hartman and Chen, among many others. Recently, Maloney, Perlmutter, Zadeh, and I developed an analytic bootstrap technique for computing Rényis and, more generally, higher-genus partition functions [11]. Using this method we were able to disprove a conjecture by Chen, Long, and Zhang about the large-c behavior of these quantities [12].

Aside from its calculational utility, the RT formula is a fundamental and deep statement about the nature of spacetime in quantum gravity. Yet its meaning remains mysterious. For example, how should one think about the bulk minimal surface whose area gives the EE of a given boundary region? A naive interpretation is that it is the locus in the bulk where, in some sense, the bits “live.” However, this idea is difficult to reconcile.

To put my work in context, I will briefly describe a couple of papers that precede the 5-year time frame specified in the application instructions.
with the tendency of the minimal surface to jump under deformations of the boundary region, and with the fact that basic quantities and properties like the mutual information [8] and SSA [2] involve subtracting the areas of surfaces passing through different parts of the spacetime. Recently, Freedman and I have shown how to solve these conceptual difficulties by reformulating RT in a way that does not refer to the minimal surface at all [13]. Instead, entanglement is represented holographically by so-called “bit threads.” These Planck-thickness filaments in the bulk connect entangled parts of the boundary, such that the EE of any region is simply the maximum number of threads that can be attached to it. Phase transitions, the mutual information, SSA, and so on all have clear interpretations in this language that correspond directly to their information-theoretic meanings. I will discuss bit threads further, including their potential implications, applications, and extensions, in my Proposed Research Plan.

The RT formula applies to static holographic systems. In the dynamical case, it is supplanted by the Hubeny-Rangamani-Takayanagi (HRT) formula, which gives the boundary EE in terms of the area of a certain extremal bulk surface [14]. Testing HRT and understanding its properties are considerably more challenging than for RT, as one must contend with the full dynamics of GR. In turn, quantum information theory implies, via HRT, some fairly non-trivial GR conjectures. A prime example here is again SSA, which translates into a statement about the areas of various spacelike surfaces, with a similar flavor to the second law of black-hole mechanics. Callan, He, and I showed in examples that HRT does obey SSA, as long as the bulk spacetime obeys the null energy condition [15]. Motivated by this work, Wall subsequently used a “maximin” reformulation of HRT to find a general argument for this statement [16].

Time dependence also raises new issues that do not occur in the static case. An example is causality. In any relativistic QFT, causality constrains what perturbations may influence the EE of a given region; in particular, a perturbation entirely within its past domain of dependence will unitarily transform the reduced density matrix and therefore not affect the EE. Hubeny, Lawrence, Rangamani, and I elucidated these constraints and, via HRT, translated them into a GR statement, which we proved [17]. This is a non-trivial check on HRT, because the extremal surface can probe behind both apparent and event horizons; it can even lie in the “causal shadow,” the region out of causal contact with the entire boundary. HRT thus teaches us that, despite being inaccessible, this region plays an important role in encoding the state of the boundary. We thus see in yet another way how quantum information theory is built into the fabric of GR.

For three-dimensional bulk spacetimes, the HRT formula implies a very general relation between the area of an almost arbitrary spacelike bulk surface and the so-called “differential entropy,” derived from the EEs of a certain family of boundary regions. This “hole-ographic” relation, which greatly generalizes the usual area-entropy relation, was first discovered by Balasubramanian et al. through the study of simple examples in a fairly restricted context (constant-time slices of $\text{AdS}_3$) [18]. Myers, Wien, and I gave a general proof of the relation, which showed that it applies in much more general settings than previously suspected, including arbitrary time-dependent three-dimensional spacetimes, higher-dimensional ones with a sufficient degree of symmetry, and even certain higher-curvature bulk gravity theories [19]. This has motivated quite a bit of subsequent work, such as the elucidation of the information-theoretic meaning of the differential entropy [20].

I have also studied entanglement in non-holographic QFTs, including both its properties and calculational methods. One important issue is whether entanglement (Rényi) entropies intrinsically characterize field theories: Are they the same for theories that are dual to each other, and different for theories that are not? Lawrence, Roberts, and I addressed this question in the context of free two-dimensional CFTs, whose duality relations are well-understood yet for which contradictory statements had appeared in the literature, for example implying that dual theories have different Rényis [21]. Through a careful analysis, we clarified the situation and established that in this context the Rényis do indeed provide an intrinsic characterization.

In three dimensions, the disk EE provides the only known renormalization-group monotone; this is the F-theorem [22]. The pure compact Maxwell theory, which is free yet undergoes a non-trivial RG flow, presents a rare opportunity to compute this quantity explicitly, as Agón, Jafferis, Kasko, and I did [23]. The calculation revealed a novel interplay between spontaneous symmetry breaking and the replica trick, and also led to a better understanding of the structure of the reduced density matrix in field theories, especially in the presence of gauge symmetries. On a technical level, we developed a numerical method to analytically continue the Rényi and find the von Neumann entropy, overcoming a persistent obstacle in the use of the replica trick. Our method was recently applied by De Nobili, Coser, and Tommi to free two-dimensional CFTs [24], where the Rényis have been known for a long time but the analytic continuation was not previously feasible.
References


