Holography & entanglement

Matthew Headrick
Brandeis University

Entanglement:

- quantum correlations between different parts of a system, e.g.

$$|\psi\rangle = \frac{1}{2^{1/2}} \left(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle \right)$$

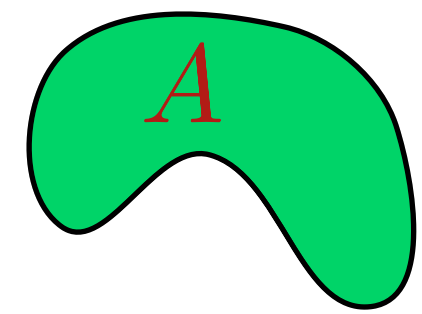
- central concept in quantum information theory & quantum computation
- increasingly important role in study of condensed-matter systems, for understanding spatial correlations at quantum level
- can be quantified in terms of entanglement entropy, mutual information, and other measures
- notoriously difficult to calculate in practice

Definitions:

- Density matrix ρ (could be pure state, e.g. $\rho = |0\rangle\langle 0|$)

- Reduced density matrix, where A is subsystem, e.g. spatial region:

$$\rho_A = \mathrm{Tr}_{A^c}\,\rho$$

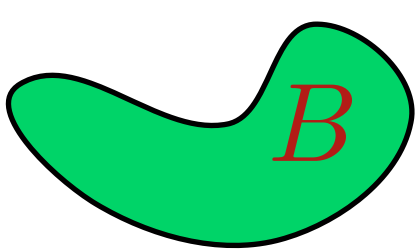


(effective density matrix if you can only observe A)

- Entanglement entropy:

$$S_A = -\,\mathrm{Tr}\,\rho_A \ln \rho_A$$

- Mutual information (measures correlations between A & B):



$$I_{A,B} = S_A + S_B - S_{A\cup B}$$

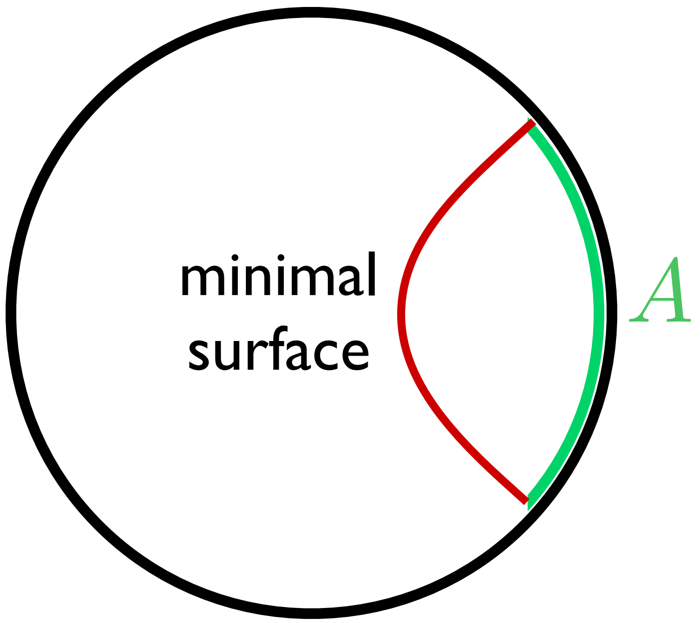
In general relativity, there is a simple & universal (but still mysterious) relation between surface area and entropy:

$$S = \frac{a}{4G_{\mathrm{N}}\hbar}$$

Example: Bekenstein-Hawking black-hole entropy

Ryu & Takayanagi (2006) proposed, for holographic field theories (in static states):

$$S_A = \frac{a_{\min}}{4G_{\mathrm{N}}\hbar}$$

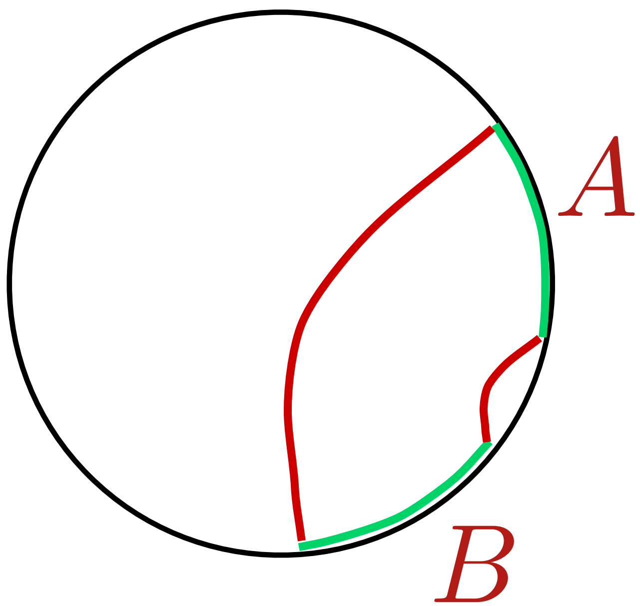


Passes many non-trivial checks, e.g. satisfies strong subadditivity (MH & Takayanagi 2007)

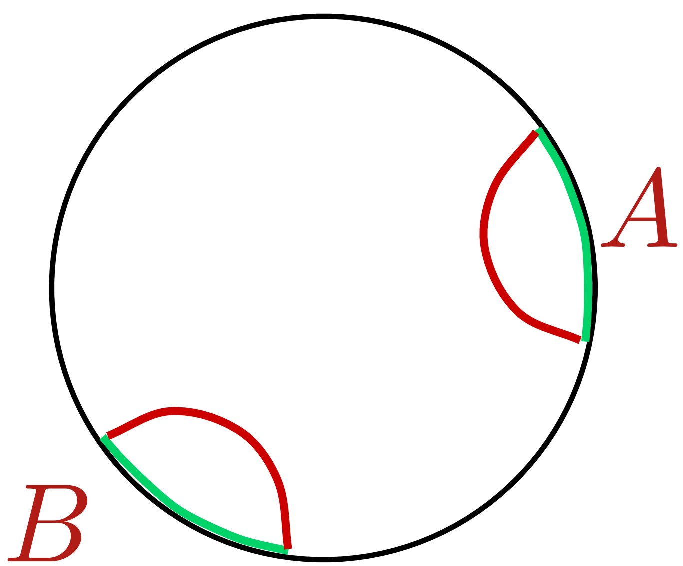
$$S_{A\cup B\cup C} + S_B \leq S_{A\cup B} + S_{B\cup C}$$

Ryu-Takayanagi formula predicts striking phenomena, for example: Mutual information has a phase transition, vanishes for separated regions (MH 2010)

$$I_{A,B} = S_A + S_B - S_{A\cup B}$$



$$\neq 0$$



$$= 0$$

More challenging:
time evolution of
entanglement entropy . . .