MATH 101A: HOMEWORK

4. Homework 4

The following problems are due Wed, Oct 3. But you can put then in my mailbox on Friday. (I return Saturday.) I will post the answers after I grade it.

4.1. Prove the properties of the pull-back, namely, if P is the pull-back of the diagram

$$G \to K \leftarrow H$$

then

(1) The kernel of $f : G \to K$ is isomorphic to the kernel of $p_2 : P \to H$.
(2) If $H \to K$ is onto, there is a bijection $H/p_2(P) \cong K/f(G)$.

4.2. Define the arrow category $\mathcal{A}r(\mathcal{G}ps)$ and show that the kernel

$$(f : G \to H) \mapsto \ker f$$

is a functor

$$\ker : \mathcal{A}(\mathcal{G}ps) \to \mathcal{G}ps.$$

Can you find the adjoint functor?

4.3. Suppose that p is prime and n is a positive integer relatively prime to p. Then show that $\mathbb{Z}_p$ is uniquely divisible by n in the sense that for any $x \in \mathbb{Z}_p$ there exists a unique $y \in \mathbb{Z}_p$ so that $ny = x$ where ny is defined to be $y + y + \cdots + y$ (n times). You may use the fact that n, p^i relatively prime implies there exist integers a_i, b_i so that

$$a_i p^i + b_i n = 1.$$

Don't use the ring structure of $\mathbb{Z}_p$. Use just the additive groups structure (as the inverse limit of the additive groups $\mathbb{Z}/p^i$).

Date: September 26, 2007.