MATH 101A: HOMEWORK

6. Homework 6

The following problems are due next Thursday (Oct 25). I will post the answers after I grade it.

All rings are commutative.

6.1. (Ex. 3 on p. 115) Show that the localization of a ring at a prime ideal is a local ring.

6.2. If R is a domain, show that the ideal in $R[X, Y]$ generated by $X^2 - Y^3$ is prime. [Hint: Consider the ring homomorphism $\phi : R[X, Y] \to R[T]$ given by $\phi(X) = T^3, \phi(Y) = T^2$.]

6.3. (Ex. 9 on p. 115) If $i = \sqrt{-1}$ show that $\mathbb{Z}[i]$ is a PID and hence a UFD. What are the units?

6.4. Do Ex. 10 on page 115, except replace 10(d) with the problem: Show that $2, 3, 1 \pm \sqrt{-5}$ are irreducible in $\mathbb{Z}[\sqrt{-5}]$.

Date: October 20, 2007.