MATH 101A: HOMEWORK

7. Homework 7

The following problems are due next Thursday (Nov 8). I will post the answers after I grade it.

Rings are allowed to be noncommutative.

7.1. If M is an R-module then show that:

(a) The set of R-module homomorphisms $f : M \to M$ is a ring under *reverse composition*:

$$fg = g \circ f$$

This ring is called $\text{End}_R(M)^{op}$.

(b) In the case of the free R-module $M = R^n$, show that there is an isomorphism of rings

$$\text{End}_R(R^n)^{op} \cong M_n(R)$$

7.2. Let $A \subseteq B \subseteq M$ be submodules. Suppose that A is pure in M and B/A is pure in M/A. Then is B pure in M?

7.3. Let R be a PID which is not a field. Let $Q(R)$ be the field of fractions of R considered as an R-modules. [Think of the example $R = \mathbb{Z}$, $Q(R) = \mathbb{Q}$.]

(a) Show that $Q(R)$ is torsion-free.

(b) Show that $Q(R)$ is not isomorphic to a submodule of R^n for any n.

7.4. Find an example of a module M which is not Noetherian but every proper submodule is cyclic.

Date: November 2, 2007.