

MATH 101A: ALGEBRA I
PART A: GROUP THEORY

In our unit on group theory we studied solvable and nilpotent groups paying particular attention to p -groups. There was also a heavy emphasis on categorical notions such as adjoint functors, limits and colimits.

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