

MATH 101A: ALGEBRA I
PART B: RINGS AND MODULES

In the unit on rings, I explained category theory and general rings at the same time. Then I talked mostly about commutative rings. In the unit on modules, I again mixed category theory into the basic notions and progressed to the structure theorem for finitely generated modules over PID's. Jordan canonical forms were used as an application. The uniqueness part of the structure theorem was put off for the discussion on tensor product in Part C.

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