

MATH 101B: HOMEWORK

1. HOMEWORK 01

The following three problems are due next Thursday (1/25/7).

1.1. Show that the category of finite abelian groups contains no non-trivial projective or injective objects. (Use the Fundamental Theorem: all finite abelian groups are direct sums of cyclic p -groups \mathbb{Z}/p^n .)

1.2. Show that abelian categories have push-outs and pull-backs. I.e., Given an abelian category \mathcal{A} and morphisms $f : A \rightarrow C, g : B \rightarrow C$ there exists an object D (called the *pull-back*) with morphisms $\alpha : D \rightarrow A, \beta : D \rightarrow B$ forming a commuting square ($f \circ \alpha = g \circ \beta$) so that for any other object X with maps to A, B forming another commuting square, there exists a unique morphism $X \rightarrow D$ making a big commutative diagram. (Draw the diagram.) Hint: There is an exact sequence

$$0 \rightarrow D \rightarrow A \oplus B \rightarrow C$$

1.3. Let k be a field and let R be the polynomial ring $R = k[X]$. Let Q be the k vector space of all sequences:

$$(a_0, a_1, a_2, \dots)$$

where $a_i \in k$ with the action of $X \in R$ given by shifting to the left:

$$X(a_0, a_1, a_2, \dots) = (a_1, a_2, a_3, \dots)$$

Then show that Q is injective. Hint: first prove that any homomorphism $f : A \rightarrow Q$ is determined by its first coordinate.