MATH 101B: HOMEWORK

1. Homework 01

The following three problems are due next Thursday (1/25/7).

1.1. Show that the category of finite abelian groups contains no non-trivial projective or injective objects. (Use the Fundamental Theorem: all finite abelian groups are direct sums of cyclic $p$-groups $\mathbb{Z}/p^n$.)

1.2. Show that abelian categories have push-outs and pull-backs. I.e., Given an abelian category $\mathcal{A}$ and morphisms $f : A \to C, g : B \to C$ there exists an object $D$ (called the pull-back) with morphisms $\alpha : D \to A, \beta : D \to B$ forming a commuting square ($f \circ \alpha = g \circ \beta$) so that for any other object $X$ with maps to $A, B$ forming another commuting square, there exists a unique morphism $X \to D$ making a big commutative diagram. (Draw the diagram.) Hint: There is an exact sequence

$$0 \to D \to A \oplus B \to C$$

1.3. Let $k$ be a field and let $R$ be the polynomial ring $R = k[X]$. Let $Q$ be the $k$ vector space of all sequences:

$$(a_0, a_1, a_2, \cdots)$$

where $a_i \in k$ with the action of $X \in R$ given by shifting to the left:

$$X(a_0, a_1, a_2, \cdots) = (a_1, a_2, a_3, \cdots)$$

Then show that $Q$ is injective. Hint: first prove that any homomorphism $f : A \to Q$ is determined by its first coordinate.

Date: January 18, 2007.