MATH 101B: HOMEWORK

2. Homework 02

The following three problems are due next Thursday (2/2/7). The strict deadline is 1:30pm Friday.

2.1. Give a precise description of the ring R indicated below and show that it has the property that left R-modules are the same as chain complexes of abelian groups with three terms, i.e.,

$$C_2 \xrightarrow{d_2} C_1 \xrightarrow{d_1} C_0$$

where C_1, C_2, C_3 are abelian groups and d_1, d_2 are homomorphisms so that $d_1 \circ d_2 = 0$ and a homomorphism of R-modules is a chain map $f : C \to D$, i.e. it consists of three homomomorphisms $f_i : C_i \to D_i$ so that the following diagram commutes.

$$\begin{array}{ccccc} C_2 & \longrightarrow & C_1 & \longrightarrow & C_0 \\ {\scriptstyle f_2}\downarrow & & {\scriptstyle f_1}\downarrow & & {\scriptstyle f_0}\downarrow \\ D_2 & \longrightarrow & D_1 & \longrightarrow & D_0 \end{array}$$

R is a quotient ring:

$$R = \begin{pmatrix} \mathbb{Z} & \mathbb{Z} & \mathbb{Z} \\ 0 & \mathbb{Z} & \mathbb{Z} \\ 0 & 0 & \mathbb{Z} \end{pmatrix} / \begin{pmatrix} 0 & 0 & \mathbb{Z} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

[Hint: R is additively free abelian with five generators, three of which are idempotents e_0, e_1, e_2 and $C_i = e_i C$.]

2.2. Describe the chain complex corresponding to the free R-module R^n (n finite).

2.3. Assume without proof that the analogous statements hold for right R-modules, namely they are cochain complexes

$$C^2 \leftarrow C^1 \leftarrow C^0$$

where $C^i = Ce_i$ (just as $C_i = e_i C$). Give a description (as a chain complex with three terms) of the injective R-module $\mathrm{Hom}_{\mathbb{Z}}(R^n_R, \mathbb{Q}/\mathbb{Z})$.

Date: January 25, 2007.