

MATH 101B: HOMEWORK

3. HOMEWORK 03

The following problem is due next Thursday (2/8/7). The strict deadline is 1:30pm Friday.

Compute $\text{Ext}_{\mathbb{Q}[X]}^i(\mathbb{Q}[X]/(f), \mathbb{Q}[X]/(g))$ using both the projective resolution of $\mathbb{Q}[X]/(f)$ and the injective coresolution of $\mathbb{Q}[X]/(g)$.

[First take the projective resolution P_* of $\mathbb{Q}[X]/(f)$. Then

$$\text{Ext}_{\mathbb{Q}[X]}^i(\mathbb{Q}[X]/(f), \mathbb{Q}[X]/(g)) \cong H^i(\text{Hom}_{\mathbb{Q}[X]}(P_*, \mathbb{Q}[X]/(g)))$$

Then, find the injective (co)resolution Q_* of $\mathbb{Q}[X]/(g)$. The Ext groups can also be found by the formula

$$\text{Ext}_{\mathbb{Q}[X]}^i(\mathbb{Q}[X]/(f), \mathbb{Q}[X]/(g)) \cong H^i(\text{Hom}_{\mathbb{Q}[X]}(\mathbb{Q}[X]/(f), Q_*))$$

The theorem that says that these two definitions are equivalent, we don't have time to prove. But working out this example might give you an idea on why it is true for $i = 0, 1$.]