MATH 101B: HOMEWORK

4. Homework 04

The following problems are due Thursday (3/1/7). The strict deadline is 1:30pm Friday.

(1) Show that unique factorization domains (UFDs) are integrally closed.
(2) Show that the integral closure of $\mathbb{Z}[\sqrt{5}]$ (in its fraction field) is $\mathbb{Z}[\alpha]$ where
$$\alpha = \frac{1+\sqrt{5}}{2}$$
(3) Combining these we see that $\mathbb{Z}[\sqrt{5}]$ is not a UFD. Find a number which can be written in two ways as a product of irreducible elements. [Look at the proof of problem 1 and see where it fails for the element α in problem 2.]
(4) If E is a finite separable extension of K and $\alpha \in E$ show that the trace $\mathrm{Tr}_{E/K}(\alpha)$ is equal to the trace of the K linear endomorphism of E given by multiplication by α. [Show that the eigenvalues of this linear transformation are the Galois conjugates of α and each eigenvalue has the same multiplicity.]

Date: February 15, 2007.