MATH 101B: HOMEWORK

4. Homework 04

The following problems are due Thursday (3/1/7). The strict deadline is 1:30pm Friday.

(1) Show that unique factorization domains (UFDs) are integrally closed.

(2) Show that the integral closure of \( \mathbb{Z}[\sqrt{5}] \) (in its fraction field) is \( \mathbb{Z}[\alpha] \) where

\[
\alpha = \frac{1 + \sqrt{5}}{2}
\]

(3) Combining these we see that \( \mathbb{Z}[\sqrt{5}] \) is not a UFD. Find a number which can be written in two ways as a product of irreducible elements. [Look at the proof of problem 1 and see where it fails for the element \( \alpha \) in problem 2.]

(4) If \( E \) is a finite separable extension of \( K \) and \( \alpha \in E \) show that the trace \( \text{Tr}_{E/K}(\alpha) \) is equal to the trace of the \( K \) linear endomorphism of \( E \) given by multiplication by \( \alpha \). [Show that the eigenvalues of this linear transformation are the Galois conjugates of \( \alpha \) and each eigenvalue has the same multiplicity.]

Date: February 15, 2007.