6. Homework 06

The following problems are due Thursday (3/22/7). The strict deadline is 1:30pm Friday.

Assume $R, M$ are both Noetherian.

(1) Show that for any ideal $I$ in $R$ there are only finitely many minimal primes containing $I$. [Take a maximal counterexample.]

(2) Suppose that $p$ is a prime and $n > 0$. Let

$$p^{(n)}M := p^n M_p | M$$

Show that this is a $p$-primary submodule of $M$.

(3) If $\phi : R \to S$ is a homomorphism of Noetherian rings and $M$ is an $S$-module then show that

$$\text{ass}_R(M) = \phi^*(\text{ass}_S(M))$$

where $\phi^* : \text{Spec}(S) \to \text{Spec}(R)$ is the map induced by $\phi$. 

\(\text{Date: March 15, 2007.}\)