MATH 101B: HOMEWORK

7. Homework 07

The following problems are due Thursday (4/12/7). The strict deadline is 3pm Friday.

(1) Show that $\mathbb{Z}[1/p]/\mathbb{Z}$ is an Artinian $\mathbb{Z}$-module. [Show that every proper subgroup is finite.]
(2) Show that the center of the ring $Mat_n(R)$ is isomorphic to the center of R. [Show that the center $Z(Mat_n(R))$ consists of the scalar multiples aI_n of the identity matrix where $a \in Z(R)$.]
(3) Suppose that M is both Artinian and Noetherian. Then show that
   (a) $r^nM = 0$ for some $n > 0$ ($r^nM := \underbrace{rr\cdots r}_{n} M$)
   (b) $r^iM/r^{i+1}M$ is f.g. semisimple for all $i \geq 0$.
(4) Prove the converse, i.e., (a) and (b) imply that M is both Artinian and Noetherian.

Date: April 4, 2007.