These are notes for our first unit on the algebraic side of homological algebra. While this is the last topic (Chap XX) in the book, it makes sense to do this first so that grad students will be more familiar with the ideas when they are applied to algebraic topology (in 121b). At the same time, it is not my intention to cover the same material twice. The topics are

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