

MATH 101B: ALGEBRA II
PART C: SEMISIMPLICITY

We have one week to talk about semisimple rings and semisimple modules (Chapter XVII). A *semisimple R -module* is a finite direct sum of simple modules

$$M = S_1 \oplus \cdots \oplus S_n$$

and a *semisimple ring* is a ring R for which all f.g. modules are semisimple. The main reasons that I am choosing this particular topic in non-commutative algebra is for the study of representations of finite groups which we will do after the break.

If G is a finite group then a representation of G over \mathbb{C} is the same as a module over the group ring $\mathbb{C}[G]$ (also written $\mathbb{C}G$). Once we have the basic definitions it will be very easy to see that $\mathbb{C}[G]$ is a semisimple ring. This makes the representation theory of finite groups elementary.

From now on, all rings will be associative rings with $1 \neq 0$ (which may or may not be commutative) and R -module will be understood to be left R -modules.

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